# Choosing Numbers that Work
# Quick Reference Guide

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Both this guide and the accompanying paper containing derivations and explanations can be downloaded in PDF format (Adobe Acrobat Reader necessary) from:
<http://www.mrmont.com/numbers>

## 1. Vertical Circles

Problem: Choose the velocity at the bottom of a vertical circle (radius $R$, no drag or friction) so that an object does not fall out at the top; or if it is assumed that it stays in the circle, avoid requiring strange occurrences such as a negative normal forces ("sticky" surface forces), or a *push* (compression rather than tension) from a rope.

What to do: Choose a velocity equal to or greater than: . . . . . . . . $\sqrt{5Rg}$

or in MKS units on Earth, approximately equal to or greater than: . . . . . $7\sqrt{R}$

Example: For a vertical circle on Earth, radius 2.0 meters, choose a velocity at equal to or greater than: $7\sqrt{2.0}$ m/s — roughly 9.9 m/s.

## 2. Springs

Problem: For a mass, $m$, attached to a vertical spring at equilibrium and released from rest on Earth (no damping, no friction), choose a spring constant $k$ in MKS units so that the extension remains less than 1 meter.

What to do: Keep the ratio $\boldsymbol{k : m}$ equal to or greater than: . . . . . . . **20:1**

Example: To a vertical ideal spring at equilibrium is attached a 5-kg mass, which is then released from rest. To keep the extension of the spring less than 1 meter, choose a spring constant at least equal to (20) * (5) , or 100 N/m.

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Problem: For a mass, $m$, on a surface approaching a horizontal spring at an initial velocity $v$ (no damping, no friction), choose a spring constant $k$ in MKS units so that the spring compression does not exceed 1 meter.

What to do: Keep the ratio of $\boldsymbol{k : mv^2}$ equal to or greater than: . . . . . . . **1:1**

Example: A 3-kg mass on a frictionless surface is approaching an ideal spring at 5 m/s. Choose a spring constant at least equal to $(3)*(5)^2$, or 75 N/m.

## 3. Projectile Motion

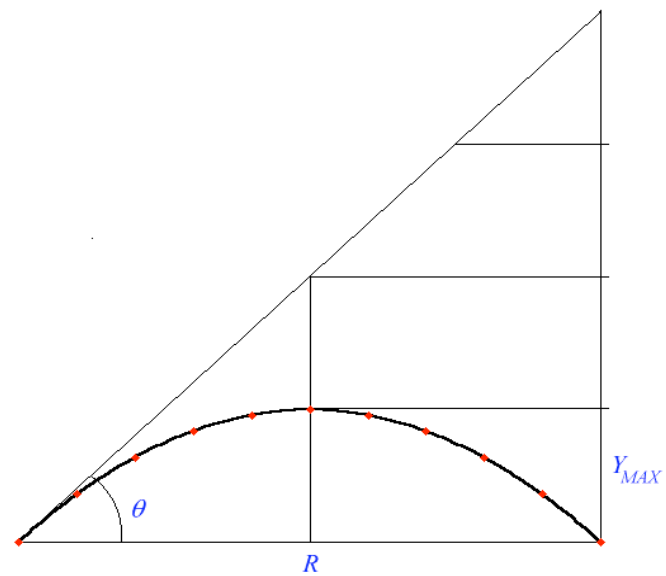
Problem: Given the desired maximum height $Y_{MAX}$ and range $R$ of a projectile launched from and landing at the same height (neglecting air resistance), find the initial launch speed $v$ and angle $\theta$.

What to do: Choose a range and a max. height, then calculate the launch angle . . $\tan\theta = \frac{4Y_{MAX}}{R}$

Then solve for the launch speed . . . . . . $v = \sqrt{\frac{Rg}{\sin 2\theta}}$ , or $\sqrt{\frac{2gY_{MAX}}{\sin^2\theta}}$

Note that the equation of the parabolic trajectory is . . . . $y = \frac{4Y_{MAX}}{R}x - \frac{4Y_{MAX}}{R^2}x^2$

Some of the relationships are neatly captured in this diagram:



## 4. Perfectly Elastic Collisions in One Dimension

Problem: Choose velocities and masses for two objects so that a linear collision between the two is perfectly elastic.

BEFORE:

AFTER:

What to do: Choose two pairs of velocities that have the same sum; Each pair will be the initial and final velocity of one object . . . . . . . . . . . $\mathbf{v}_{1i} + \mathbf{v}_{if} = \mathbf{v}_{2i} + \mathbf{v}_{2f}$

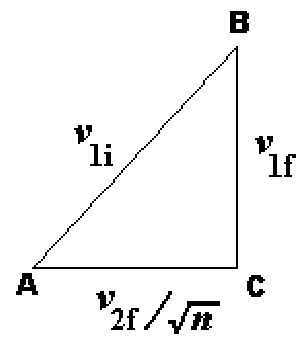
Then choose masses that satisfy the ratio . . . . . . $\dfrac{m_2}{m_1} = - \dfrac{\Delta \mathbf{v}_1}{\Delta \mathbf{v}_2}$

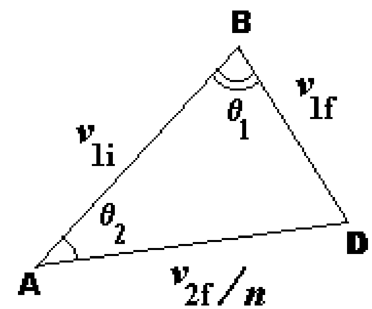
Caution: Choose initial and final values so that if one has a positive change in velocity, the other has a negative change. Also, avoid having the same initial and final velocity for an object, unless you intend that object to be infinitely more massive than the other.

Example: Choose initial and final velocity pairs such as {3, 4} and { – 9, 2}, which both have the same sum (seven, in this case). If we let $\mathbf{v}_{1i} = 3$, and $\mathbf{v}_{1f} = 4$, we have a positive change in velocity of 1. Therefore, we must have a negative change of velocity for the other object; so let $\mathbf{v}_{2i} = 2$, and $\mathbf{v}_{2f} = -9$, which is a change of – 11. The mass ratio $m_2 / m_1$ must then be $-(1 / -11)$, or 1/11. So choose, say, $m_2 = 2$ kg, and $m_1 = 22$ kg.

## 5. Perfectly Elastic Collision in Two Dimensions (Billiard Problem)

Problem: Choose initial and final speeds and angles for an elastic collision in two dimensions (one object initially at rest), without either overdetermining or underdetermining the problem. Variables: $v_{1i}$ , $v_{if}$ , $v_{2f}$ , $n$ (the ratio of the masses, $\theta_1$ , and $\theta_2$ .



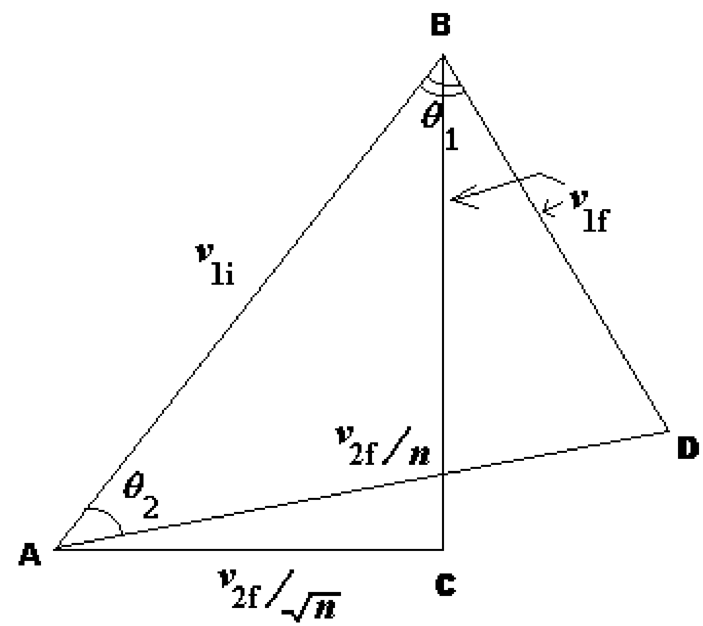




What to do: Select numbers which form the sides of the right triangle. These sides will be the magnitude of the sides of the right triangle ABC: $v_{1i}$ , $v_{if}$ and $v_{2f} / \sqrt{n}$ . Select a mass- ratio $n$ ; this ratio now determines $v_{2f}$. The law of sines or cosines for triangle ABD determines the angles $\theta_1$ and $\theta_2$

Caution: Actually, any three of the variables mentioned could be chosen first and the geometry of the situations could be used to determine the rest, EXCEPT: do not arbitrarily choose the mass-ratio $n$ , $\theta_1$ , and $\theta_2$ — they are interrelated:

~~$\tan\theta_1 = \sin 2\theta_2 /(2\sin^2\theta_2 + n - 1)$ .~~

Example: Let us use a 3-4-5 triangle for our right triangle. The hypotenuse, 5, must be $v_{1i}$, but we are free to let $v_{1f}$ be 3 or 4; we choose $v_{1f} = 3$. This leaves $v_{2f} / \sqrt{n}$ equal to 4. We now arbitrarily choose the mass-ratio $n$ to be 2. Thus $v_{2f}$ must be $4\sqrt{2}$.



Correction to the crossed out equation:

$$n = \frac{\sin^2(\theta_2 - \theta_1) - \sin^2\theta_1}{\sin^2\theta_2}$$

## 6. Parallel Circuits Problems

Problem: Choose values for resistances in a parallel circuit that yield an equivalent resistance which is a whole number.

What to do for two resistors:

Choose a ratio between the resistors . . . . . . . . . . $a : b$

The ratio will work for multiples of the sum of the ratio numbers . . . . . . . . . . $(a + b)$, or $2\,(a + b)$, or $3\,(a + b)$, or etc.

Example: Choose a ratio between the two resistors, such as 2:3. This ratio will work for multiples of (2 + 3), which is 5 — the ratio will work for multiples of 5. So you could choose sets of resistances: {10,15}, or {20, 30}, or {30, 45}, etc.

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What to do for three or more resistors:

Choose a ratio for the resistances . . . . . . . $a : b : c$

For $n$ resistances, sum up all the combinations of them taken $(n - 1)$ at a time (there will be $n$ terms to add up) . . . . $bc + ac + ab$

The ratio will work for multiples of this sum . . . . . . $(bc + ac + ab)$, or $2(bc + ac + ab)$, or $3(bc + ac + ab)$, or etc.

Example: Let us choose, say, a ratio of 1:2:3:4 for the resistances. This ratio will yield a whole number equivalent resistance for multiples of (2)(3)(4) + (1)(3)(4) + (1)(2)(4) + (1)(2)(3), which is 50. Therefore, the ratio will work for multiples of 50. So, you could choose sets of resistances: {50, 100, 150, 200}; or {100, 200, 300, 400}; etc.