2D MOTION

0 Independence of x and y

1. Position, Velocity, and Trajectories

2. Average Velocity & Average Acceleration

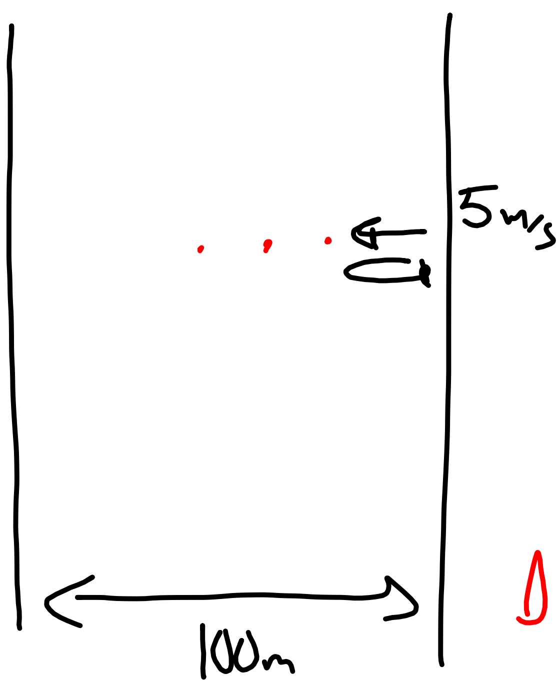
3. Instantaneous Velocity & Instantaneous Acceleration

4. Projectile Motion

5. Relative Motion

6. Example Probs

0 Independence of x and y



How long does it take to row across the river? (No current.)

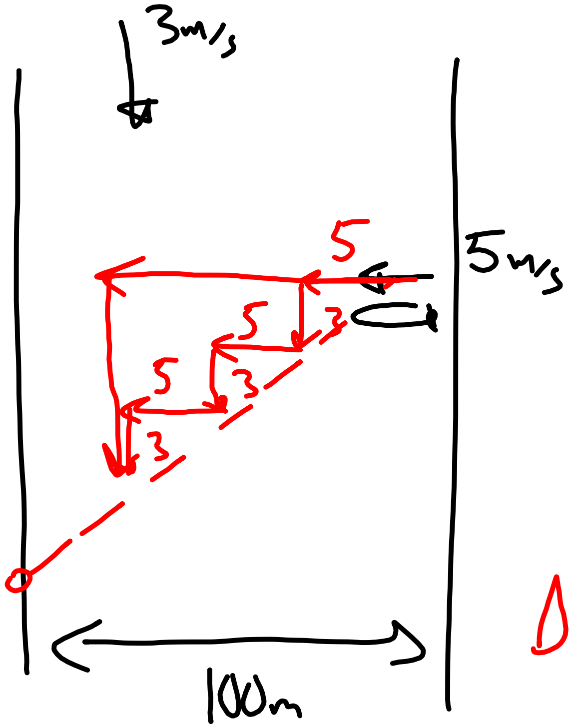
every second,

$\Delta x = -5$ m

$\Delta t = 20$ sec

0 Independence of x and y

How long does it take to row across the river? (With current.)



every second

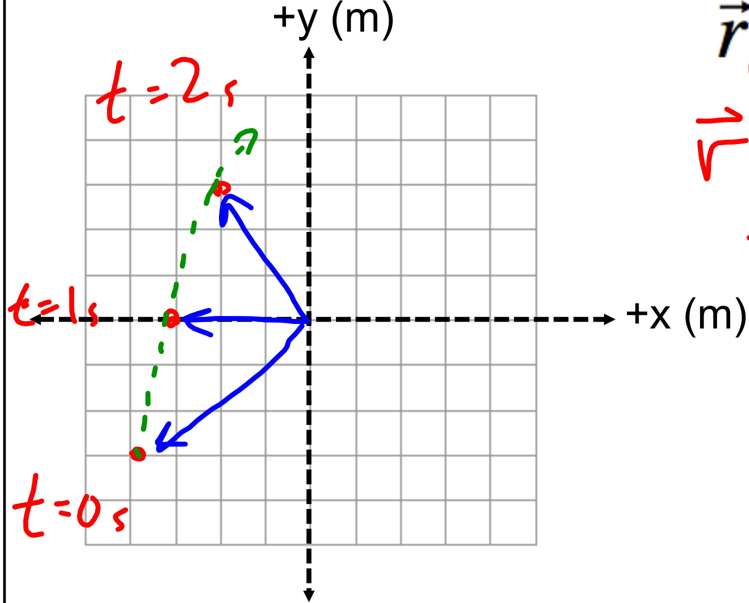
$\Delta x = -5m$ ←

$\Delta y = -3m$

$\Delta t = 20s$

1. Position, Velocity, and Trajectories

(-4, -3)

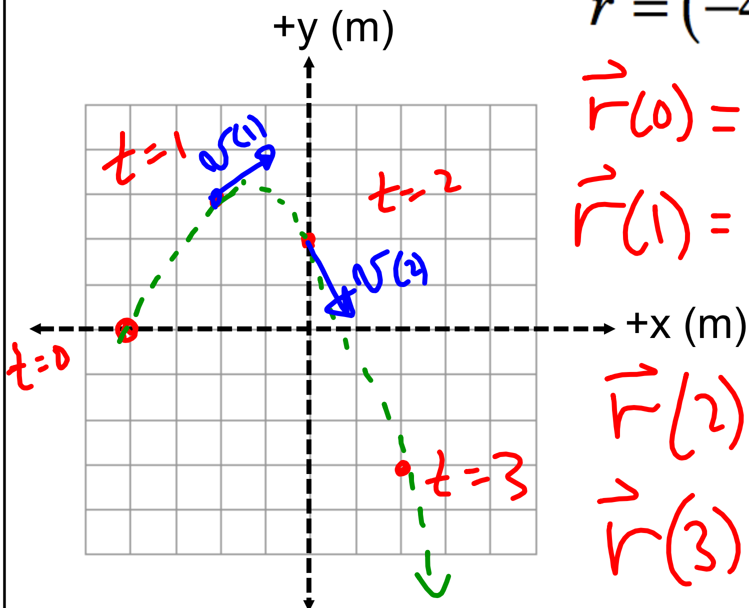


$\vec{r}(0) = -4\hat{i} - 3\hat{j}$

$\vec{r}(1) = -3\hat{i} + 0\hat{j}$

$\vec{r}(2) = -2\hat{i} + 3\hat{j}$

1. Position, Velocity, and Trajectories



$$\vec{r} = (-4 + 2t)\hat{i} + (5t - 2t^2)\hat{j}$$

$$\vec{r}(0) = -4\hat{i} + 0\hat{j}$$

$$\vec{r}(1) = -2\hat{i} + 3\hat{j}$$

$$\vec{r}(2) = 0\hat{i} + 2\hat{j}$$

$$\vec{r}(3) = 2\hat{i} - 3\hat{j}$$

2. Average Velocity & Average Acceleration

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{r} = (-4 + 2t)\hat{i} + (5t - 2t^2)\hat{j}$$

3. Instantaneous Velocity & Instantaneous Acceleration

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{r} = (-4 + 2t)\hat{i} + (5t - 2t^2)\hat{j}$$

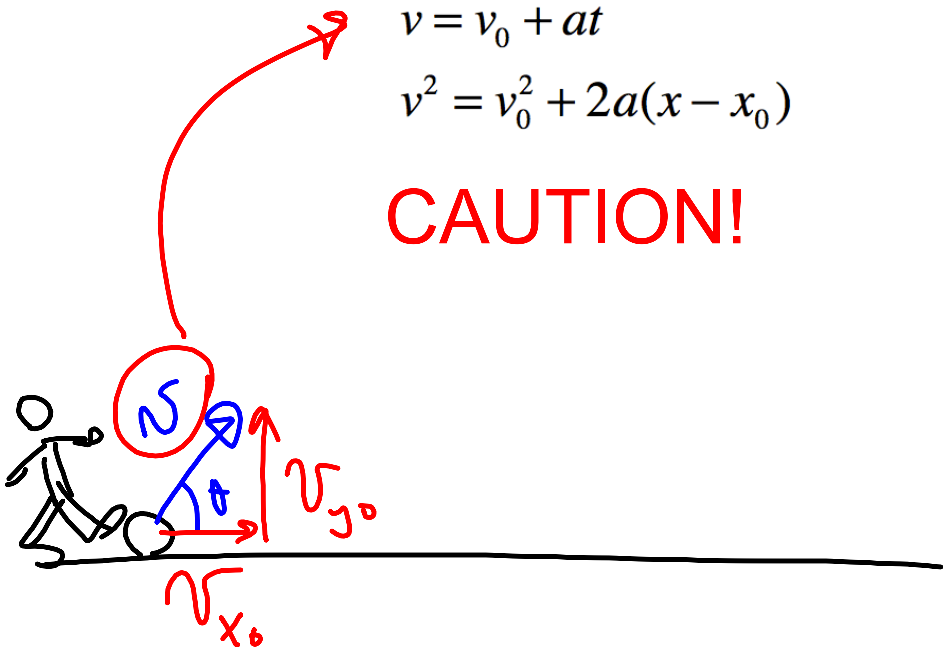
4. Projectile Motion

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

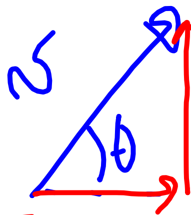
$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

CAUTION!



4. Projectile Motion



$v_{y0} = v_0 \sin\theta$

$v_{x0} = v_0 \cos\theta$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

x-direction ($a_x = 0$)

$x = x_0 + v_{x0} t$

$\Delta x = v_{x0} t$

$v_x = v_{x0}$

$v_x^2 = v_{x0}^2$

y-direction ($a_y = -10 \ \frac{m}{s^2}$)

$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$

$v_y = v_{y0} + a_y t$

$v_y^2 = v_{y0}^2 + 2 a_y \Delta y$

5. Relative Motion

v_{person} wrt H_2O?

$\vec{r} = \left[\left(2t - t^2\right)\hat{i} + \left(5 - t^2\right)\hat{j}\right] m$

Given the above position vector...

a) Draw the trajectory from t = 0 to t = 3 s.

b) Sketch the velocity vector at t = 1 s.

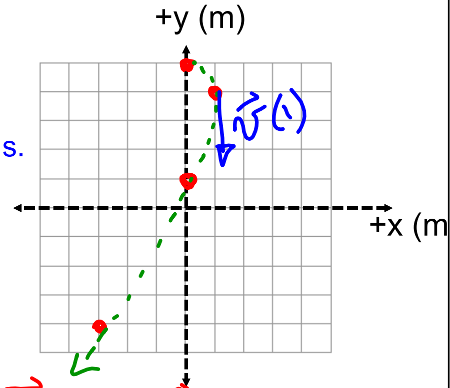
c) Find average velocity vector

between t = 0 and t = 3 s.

d) Find the velocity vector at t = 1 s.

e) Find the acceleration vector

as a function of time.



$\vec{r}(0) = 0\hat{i} + 5\hat{j}$

$\vec{r}(1) = 1\hat{i} + 4\hat{j}$

$\vec{r}(2) = 0\hat{i} + 1\hat{j}$

$\vec{r}(3) = -3\hat{i} - 4\hat{j}$

c) $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(3) - \vec{r}(0)}{3 - 0\,s}$

$= \frac{(-3\hat{i} - 4\hat{j}) - (0\hat{i} + 5\hat{j})}{3\,s}$

$= \frac{-3\hat{i} - 9\hat{j}}{3\,s} = \left(-1\hat{i} - 3\hat{j}\right)\frac{m}{s}$

d) $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[\left(2t - t^2\right)\hat{i} + \left(5 - t^2\right)\hat{j}\right]$

$\vec{v} = (2 - 2t)\hat{i} + (-2t)\hat{j}$

$= (2 - 2(1))\hat{i} + (-2(1))\hat{j}$

$= 0\hat{i} - 2\hat{j}\ \frac{m}{s}$

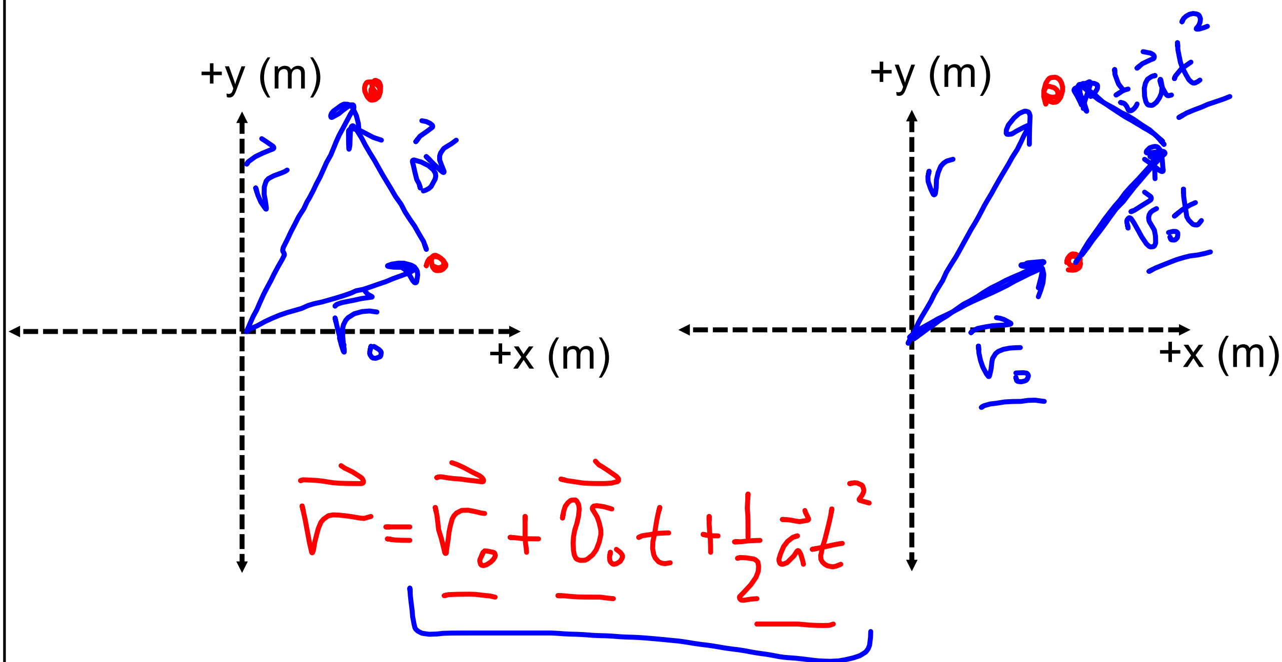
e) $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[(2 - 2t)\hat{i} + (-2t)\hat{j}\right]$

$= -2\hat{i} - 2\hat{j}\ \frac{m}{s^2}$

Thinking about equations as vector equations

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$\vec{r} = \vec{r}_0 + \Delta \vec{r}$$



A ball is thrown at 20 m/s, 60 degrees, 1.5 meters above the ground.

a) Find the baseball's max height.

b) When does it hit the ground?

c) How far did it go horizontally?

$v_{x0} = v_0 \cos\theta$

$= 20\,\frac{m}{s} \cos 60° = 10\,\frac{m}{s}$

$v_{y0} = v_0 \sin\theta$

$= 20\,\frac{m}{s} \sin 60° = 17.32\,\frac{m}{s}$

a) $v_{y0} = 17.32\,\frac{m}{s}$

$a_y = -10\,\frac{m}{s^2}$

$\Delta y = ?$

$v_y = 0\,\frac{m}{s}$

$v_y^2 = v_{y0}^2 + 2a_y\Delta y$

$0^2 = (17.32)^2 + 2(-10)\Delta y$

$0 = 300 - 20\Delta y$

$-300 = -20\Delta y$

$15\,m = \Delta y$

$+1.5\,m$

$\boxed{16.5\,m = y}$

b) $v_{y0} = 17.32\,\frac{m}{s}$

$y_0 = 1.5\,m$

$y = 0$

$a_y = -10\,\frac{m}{s^2}$

$t = ?$

$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$

$0 = 1.5 + 17.32t + \frac{1}{2}(-10)t^2$

$0 = 1.5 + 17.32t - 5t^2$

$t = -0.08\,s$ or $\boxed{3.55\,s}$

c) $\Delta x = ?$

$v_{x0} = 10\,\frac{m}{s}$

$t = 3.55\,s$

$\Delta x = v_{x0}t$

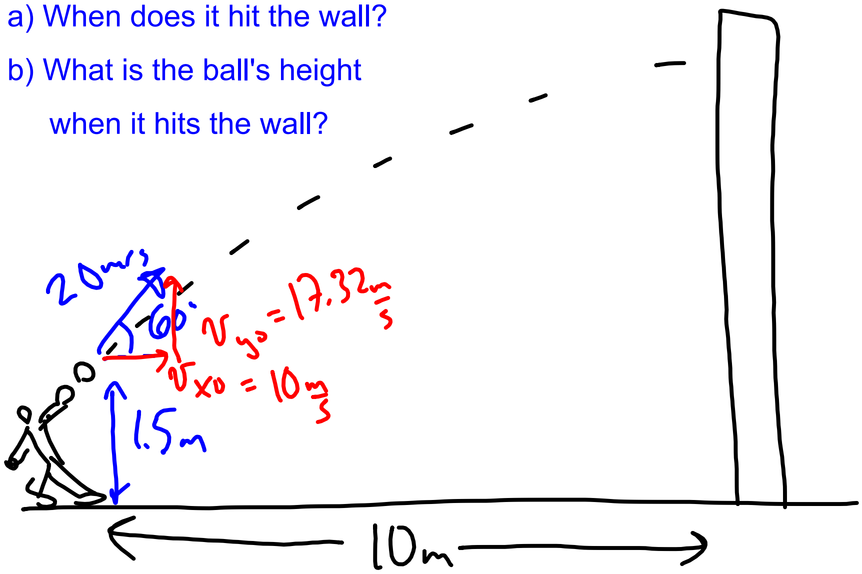
$= (10\,\frac{m}{s})(3.55\,s)$

$\boxed{= 35.5\,m}$

A ball is thrown at 20 m/s, 60 degrees, 1.5 meters above the ground.

a) When does it hit the wall?

b) What is the ball's height when it hits the wall?



a) $\Delta x = v_{x0} t$

$10\,m = (10\,\frac{m}{s})\,t$

$\boxed{1\,s = t}$

b) $\Delta y = v_{y0} t + \frac{1}{2} a_y t^2$

$\Delta y = 17.32(1) + \frac{1}{2}(-10)(1)^2$

$17.32 - 5$

$\Delta y = 12.32$

$y_0 = 1.5\,m$

$y = 12.32 + 1.5 = \boxed{13.82\,m}$

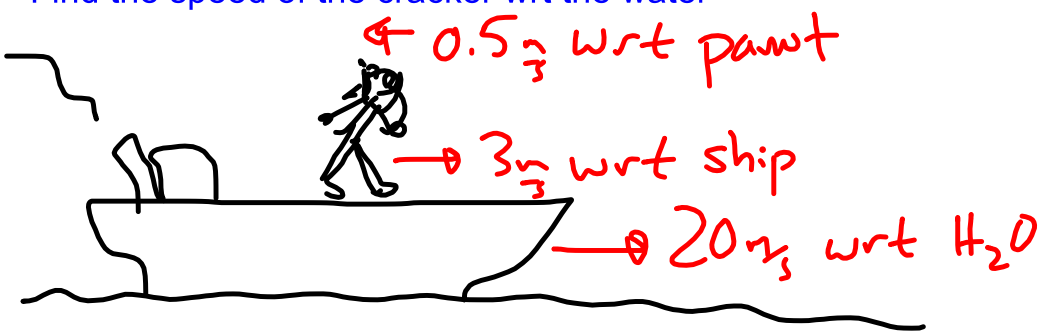
There is ship moving at 20 m/s to the right wrt the water.

On the ship there is a person walking to the right at 3 m/s wrt the ship

On the person's shoulder, there is a parrot at rest wrt the person.

The person is feeding the parrot a cracker, which is moving into the parrot's mouth at 0.5 m/s to the left wrt the parrot.

Find the speed of the cracker wrt the water



$$v_{cracker\ wrt\ H_2O} = 20\frac{m}{s} + 3\frac{m}{s} + 0\frac{m}{s} - 0.5\frac{m}{s}$$

$$= 22.5\frac{m}{s}$$