...to the new eyes of thee

All things by immortal power,

Near or far,

Hiddenly

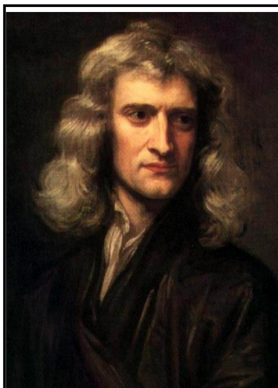
To each other linkèd are,

That thou canst not stir a flower

Without troubling of a star...

From 'The Mistress of Vision'

By Francis Thompson (1859–1907)

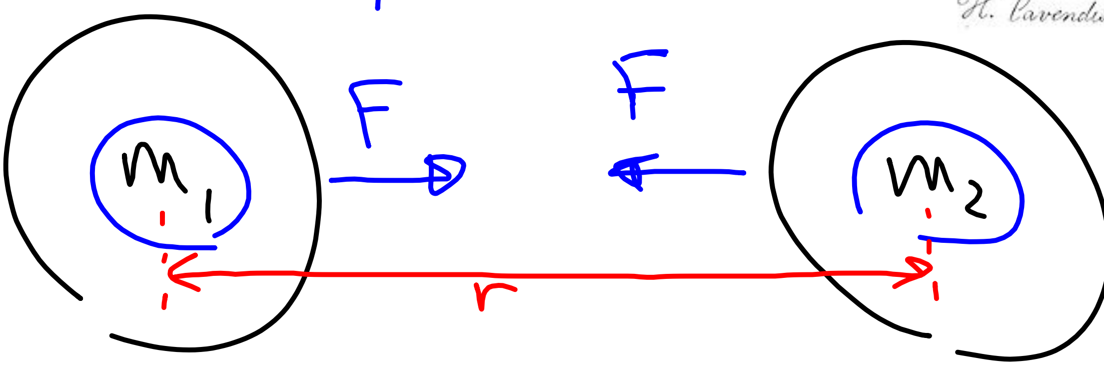


UNIVERSAL GRAVITATION

Newton & Cavendish

$$F \propto \frac{1}{r^2}$$

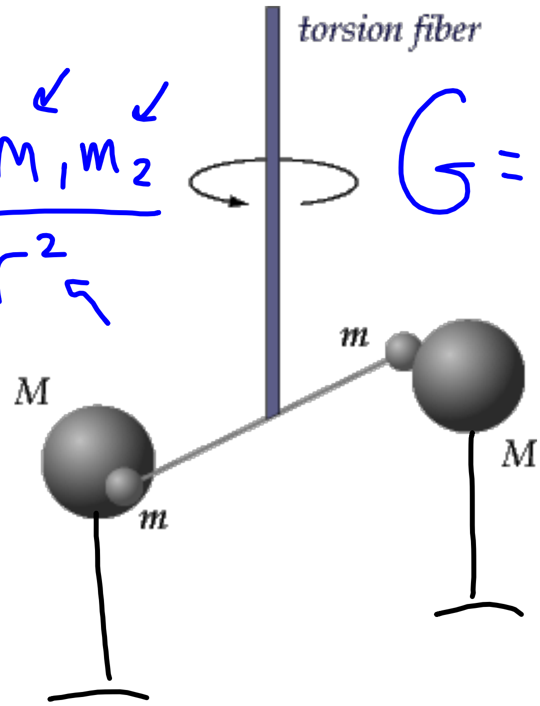
$$F \propto m_1 m_2$$



$$F \propto \frac{m_1 m_2}{r^2}$$

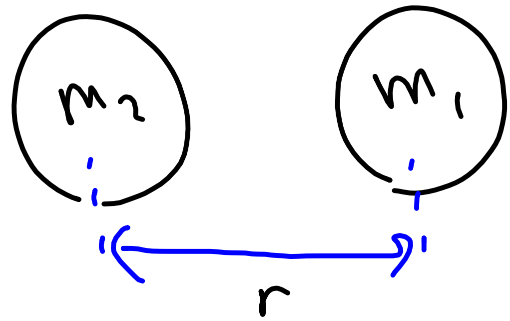
$$F = \frac{G m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$



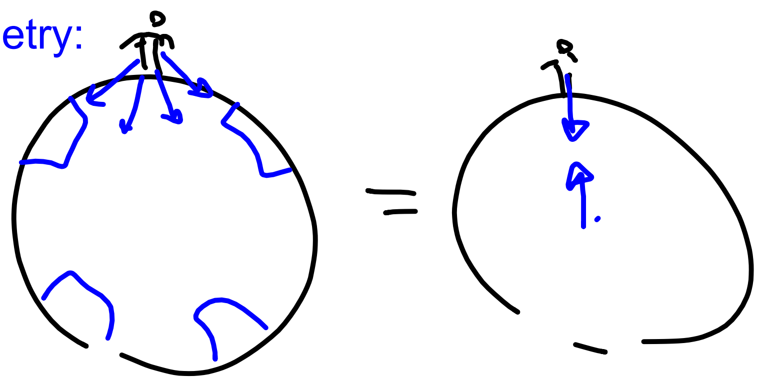
The Law of Universal Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

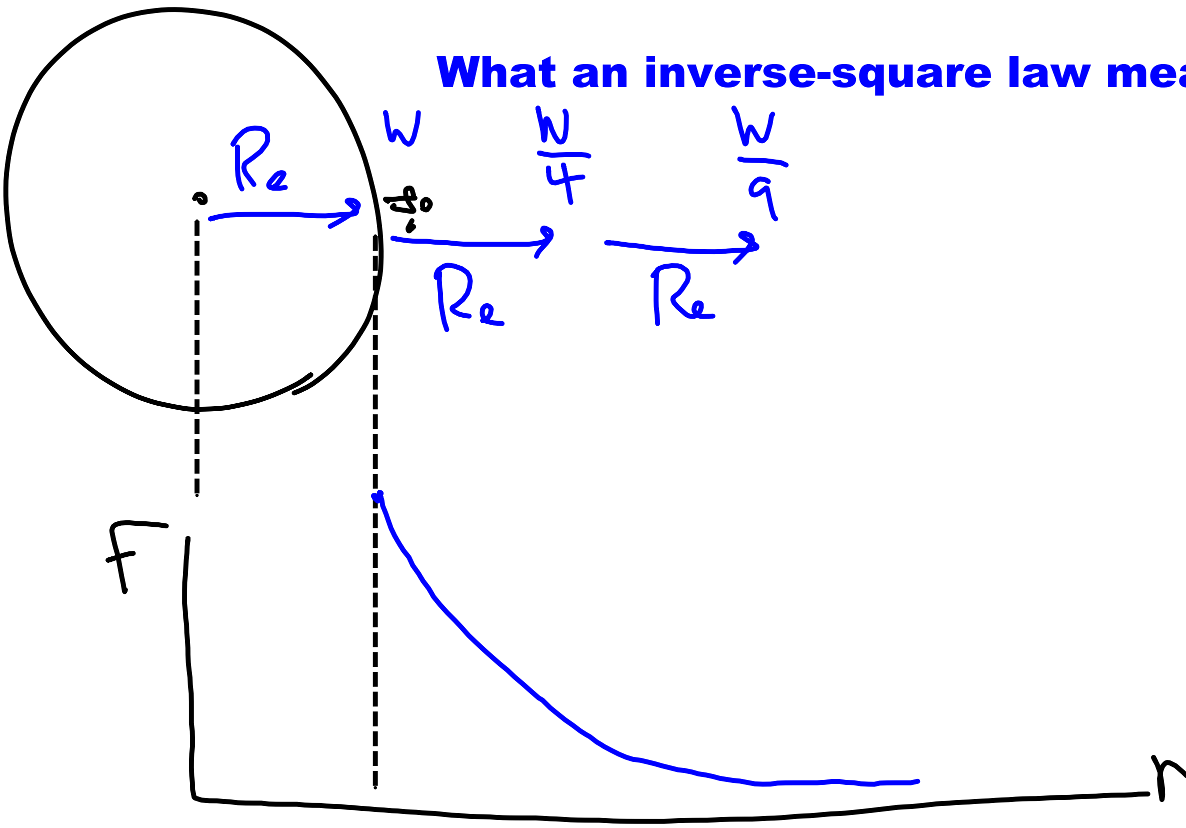


$$G = 6.67 \; x \; 10^{-11} \; Nm^2/kg^2$$

spherical symmetry:



What an inverse-square law means



Little g and Big G

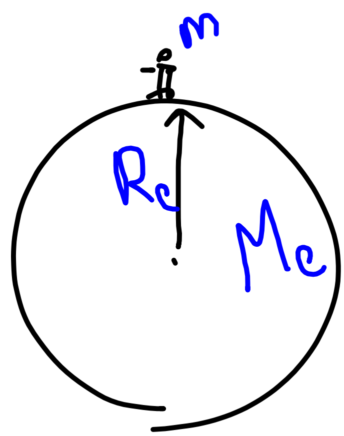
How does the universal law relate to mg?

$$F = \frac{G m_1 m_2}{r^2}$$

same

$$W = m g$$

$$F = \left(\frac{G M_e}{R_e^2}\right) m$$



$M_e = 5.98 \times 10^{24}$ kg

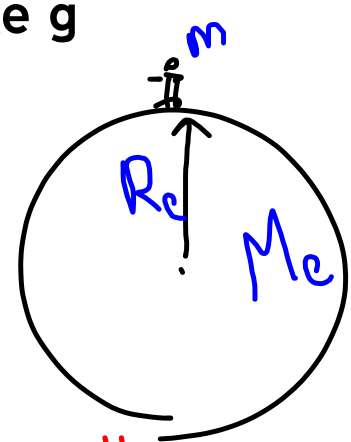
$R_e = 6.37 \times 10^{6}$ m

$$\boxed{g = \frac{G M_e}{R_e^2}}$$ earth

$$\boxed{g = \frac{G M}{r^2}}$$

Let's look at the units of little g

$$g = \frac{GM}{r^2} \qquad \frac{\left(\frac{Nm^2}{kg^2}\right)(kg)}{m^2}$$



$10 \frac{N}{kg}$ → grav field strength

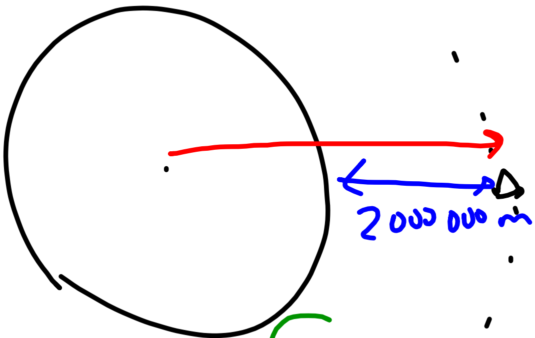
acceleration due to gravity $\equiv$ gravitational field strength

m/s^2 N/kg

A 1,000 kg satellite is 2,000 km above the surface of the Earth.
(a) Find F between Earth & satellite
(b) Find g at that distance

Me = 5.98 x 10^24 kg

Re = 6.37 x 10^6 m



$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{(6.67\times10^{-11})(5.98\times10^{24})(1000)}{(6.37\times10^6\,m + 2\times10^6\,m)^2}$$

$$= 5693\,N$$

b) $g = ?$

$$g = \frac{GM}{r^2} \qquad g = \frac{F}{m} = \frac{\frac{GMm}{r^2}}{m}$$

$$g = \frac{5693\,N}{1000\,kg} \qquad g = \frac{GM}{r^2}$$

$$\boxed{g = 5.693\,\frac{m}{s^2}}$$

A certain planet has double the mass of Earth, and double its radius - determine g at the new planet's surface.

(This can be done without using Earth's actual M and R to figure out the new planet's M and R.)

$g = 10$ $\quad g_e = \frac{GM}{r^2}$

$$g_p = \frac{G(2M)}{(2r)^2}$$

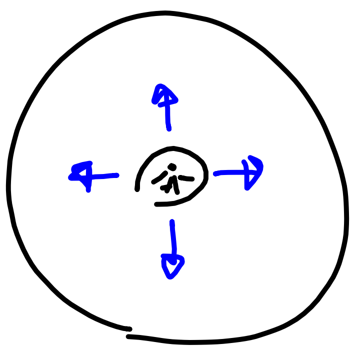
$$= \frac{G(2M)}{4r^2}$$

$$= \frac{2}{4}\frac{GM}{r^2}$$

$$= \frac{1}{2}\frac{GM}{r^2}$$

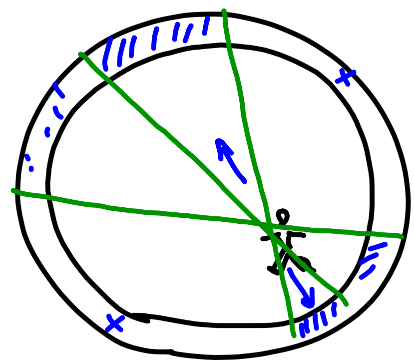
$$= \frac{1}{2}(10) = \boxed{5\,\frac{m}{s^2}}$$

What happens if you are inside a planet?



At the center?

0



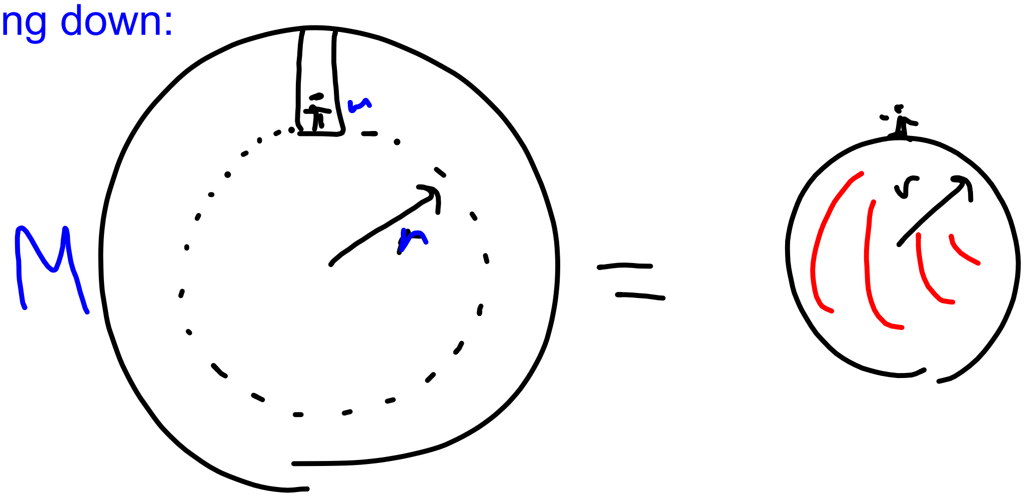
Anywhere inside a hollow planet?

0

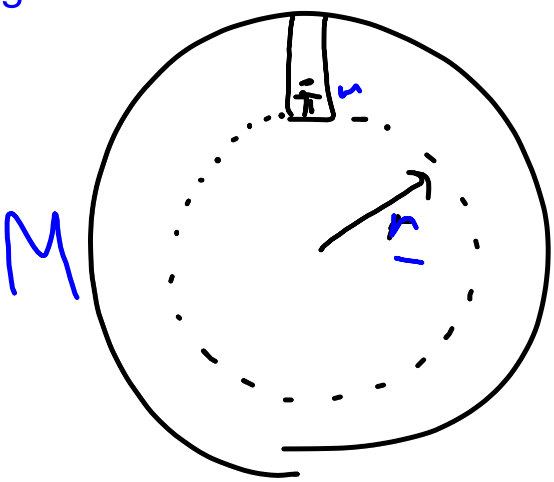
At any point inside, the amount of planet above you functions like the hollow planet idea - its gravity cancels out.

Therefore your weight is only due to the amount of planet with a smaller radius than you.

Digging down:



Digging down:



$$F = \frac{G m_1 m_2}{r^2}$$

density:

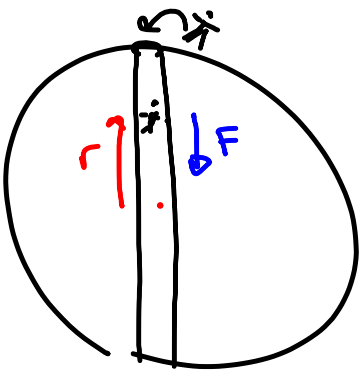
$$\rho = \frac{\text{Mass}}{\text{Volume}} \longrightarrow M_{under} = \rho V_{under}$$

$$= \rho\left(\frac{4}{3}\pi r^3\right)$$

$$F = \frac{G\left(\rho \frac{4}{3}\pi r^{\cancel{3}}\right) m}{\cancel{r^2}}$$

$$\boxed{F = \frac{4}{3} G \pi \rho m r}$$

density of planet



Strange thought: what if you dug a hole all the way through a planet and jumped in?

Would you shoot out the other side?

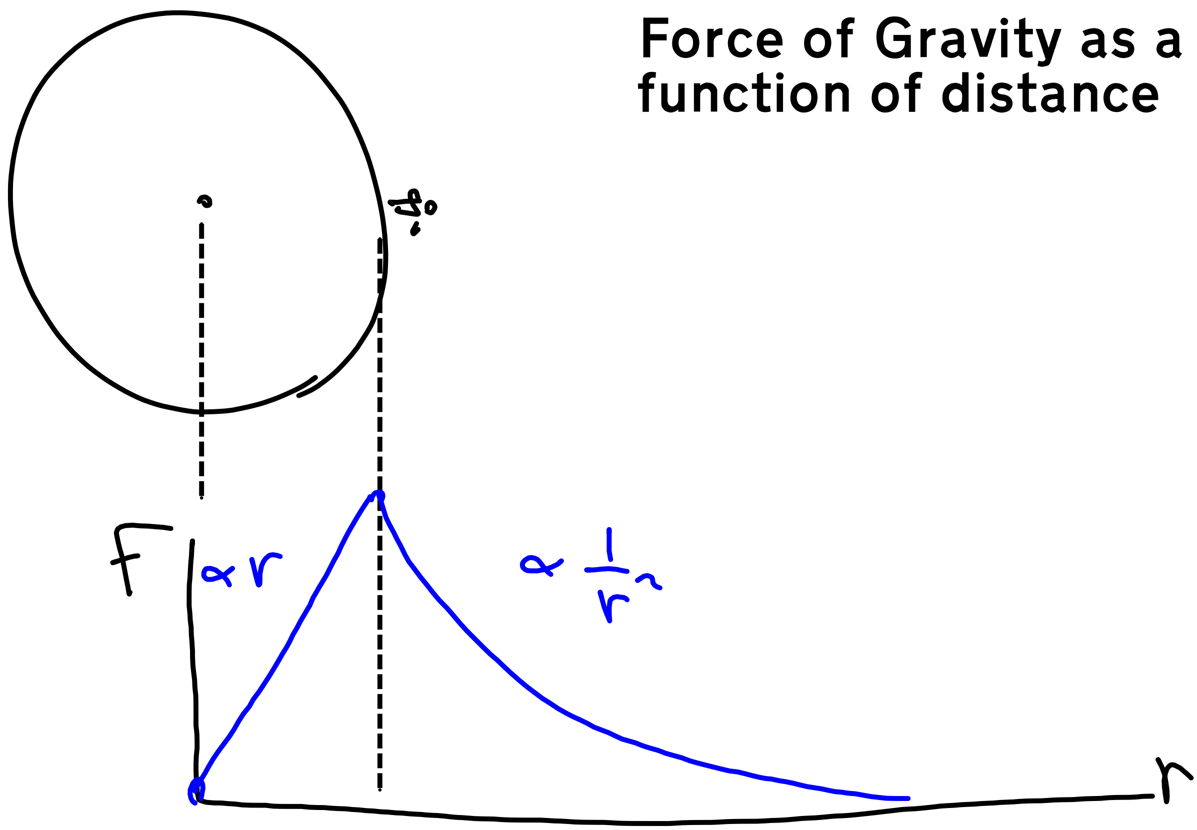
Would you stop in the middle?

???

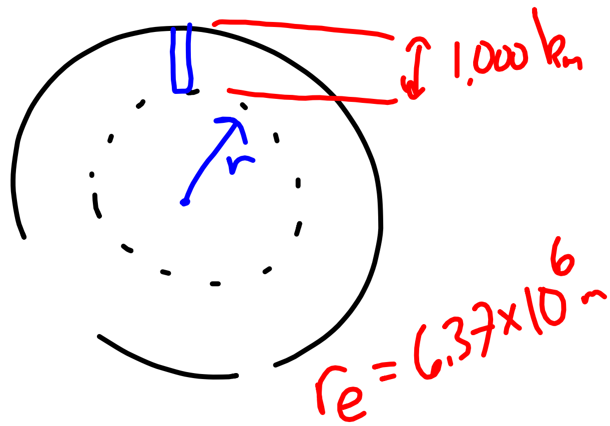
$$F = \left(-\frac{4}{3}G\pi\rho m\right)r$$

$$F = (-k)x$$

Force of Gravity as a function of distance



The Earth has an average density of 5500 kg/m³. Determine g, the acceleration due to gravity, if you were to dig down 1,000 km.



$$\cancel{F = \frac{G m_1 m_2}{r^2}}$$

$$F = \frac{4}{3} G \pi \rho m r$$

$$W = mg$$

$$\frac{W}{m} = g$$

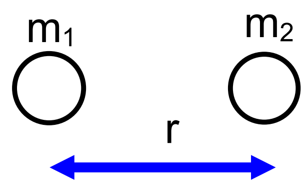
$$g = \frac{4}{3} G \pi \rho r$$

$$g = \frac{4}{3}(6.67\times10^{-11})(3.14)(5500)(6.37\times10^{6} - 1\times10^{6})$$

$$\boxed{= 8.25 \frac{m}{s^2}}$$

Gravitational Potential Energy

Can we use mgh?



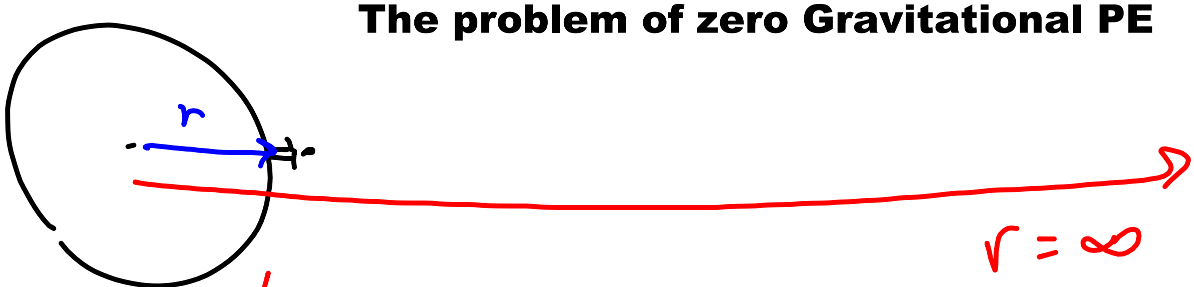
$$U = \int F\,dr = \int \frac{Gm_1m_2}{r^2}\,dr$$

$$= Gm_1m_2 \int r^{-2}\,dr$$

$$= Gm_1m_2 \left[\frac{r^{-1}}{-1}\right]$$

$$\boxed{U = -\frac{Gm_1m_2}{r}}$$

The problem of zero Gravitational PE



$r = \infty$

$U = 0$

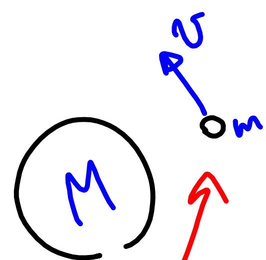
$$U = -\frac{Gm_1 m_2}{r}$$

You can't set zero grav potential wherever you want any more. Zero grav potential is at infinity.

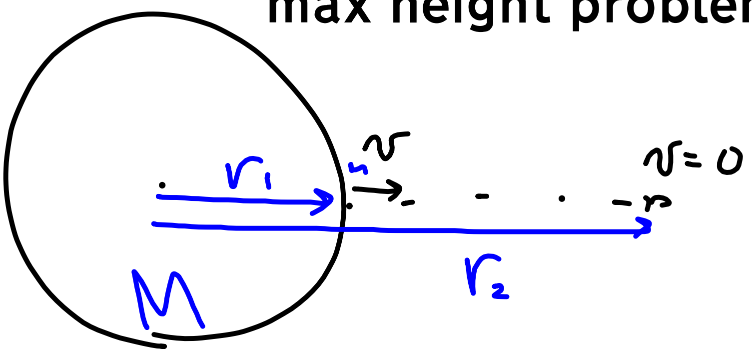
Total Energy

$$E = K + U$$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$



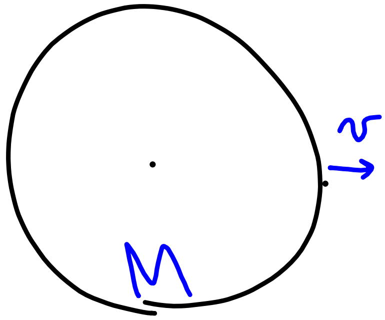
max height problems with changing g



$$E_1 = E_2$$

$$K_1 + U_1 = U_2$$

$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{r_2}$$



How fast must an object be launched to reach a height of 2,000,000 m above Earth's surface? Assume the object is launched radially outward.

Me = 5.98 x 10^24 kg

Re = 6.37 x 10^6 m

$$E_1 = E_2$$

$$\frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{r_1} = -\frac{GM\cancel{m}}{r_2}$$

$$\frac{1}{2}v^2 - \frac{GM}{r_1} = -\frac{GM}{r_2}$$

$r_1 \rightarrow$ radius of E.

$r_2 \rightarrow$ radius of E. + 2,000,000 m

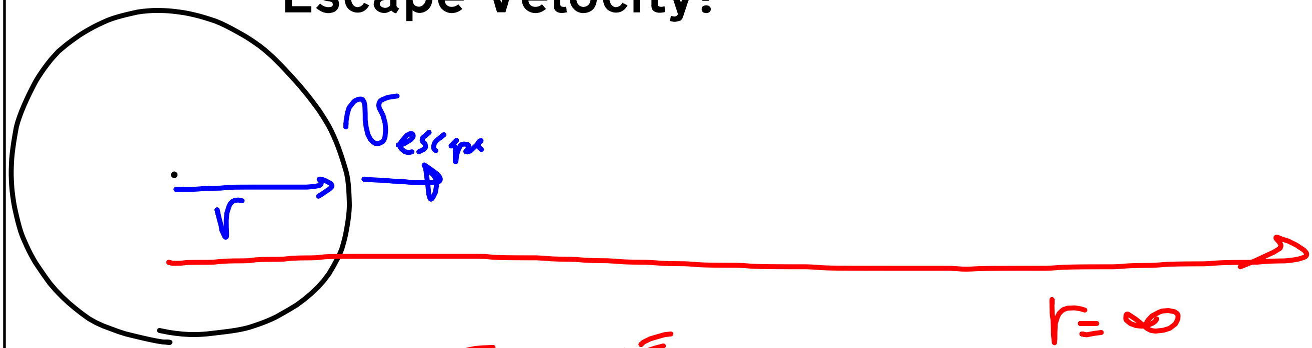
$$\frac{1}{2}v^2 = +\frac{(6.67\times10^{-11})(5.98\times10^{24})}{6.37\times10^6} - \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.37\times10^6 + 2\times10^6)}$$

$$\frac{1}{2}v^2 = 1.49\times10^7$$

$$v^2 = 2.99\times10^7$$

$$\boxed{v = 5470\,\frac{m}{s}}$$

Escape Velocity!



$$E_1 = E_2$$

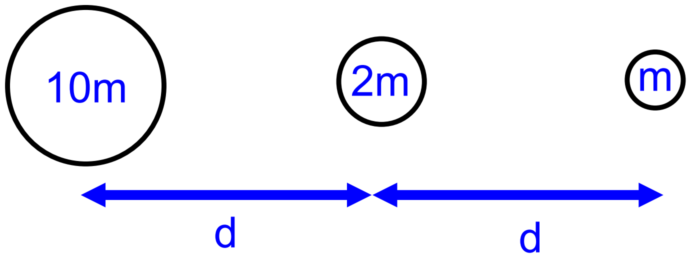
$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{\infty} + \frac{1}{2}m(0)^2$$

$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = 0$$

$$\frac{1}{2}\cancel{m}v^2 = \frac{GM\cancel{m}}{r}$$

$$\boxed{v_{escape} = \sqrt{\frac{2GM}{r}}}$$

Find U total for the system



$$U = -\frac{Gm_1 m_2}{r}$$

add up every pair.

$$-\frac{G(10m)(2m)}{d} - \frac{G(2m)(m)}{d} - \frac{G(10m)(m)}{2d}$$

$$-20\frac{Gm^2}{d} - 2\frac{Gm^2}{d} - 5\frac{Gm^2}{d}$$

$$\boxed{-27\frac{Gm^2}{d}}$$