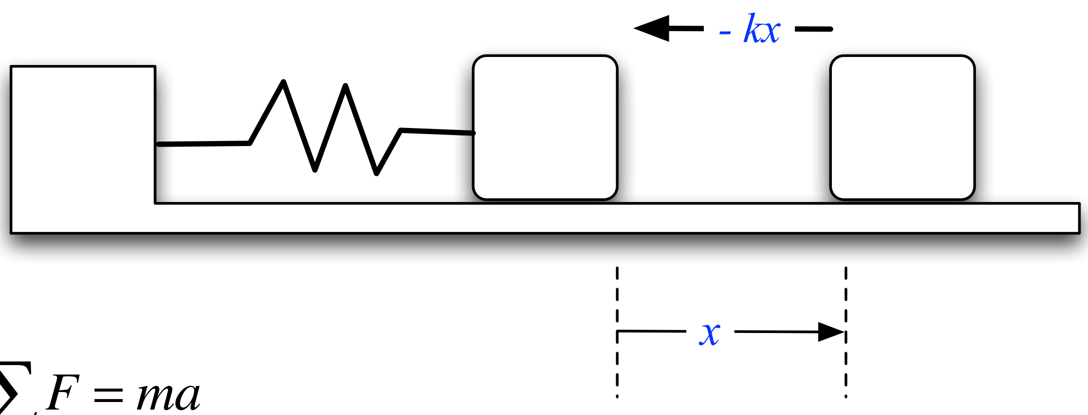
Using Differential Equations to Solve for T for a Spring System



$$\sum F = ma$$

$$-kx = ma$$

$$0 = ma + kx$$

$$0 = m\frac{d^2x}{dt^2} + kx$$

substituting $\frac{d^2x}{dt^2} = -\omega^2 x_{\max}\cos(\omega t + \phi)$ and $x = x_{\max}\cos(\omega t + \phi)$:

$$0 = -m\omega^2 x_{\max}\cos(\omega t + \phi) + kx_{\max}\cos(\omega t + \phi)$$

canceling $x_{\max}\cos(\omega t + \phi)$ from both terms:

$$0 = -m\omega^2 + k$$

solving for ω:

$$\omega = \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Using Differential Equations to Solve for T for a Spring System (Energy Approach)

Springs

$$E = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2$$

$$\frac{d}{dt}\left(E = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2\right)$$

$$0 = 2\tfrac{1}{2}kx^{2-1}\left(\frac{dx}{dt}\right) + 2\tfrac{1}{2}mv^{2-1}\left(\frac{dv}{dt}\right)$$

$$0 = kxv + mva$$

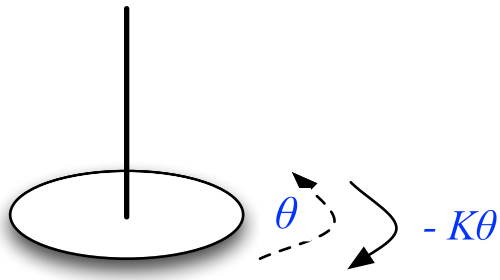
$$\left[\text{cancel out } v\right]$$

$$0 = kx + ma$$

$$0 = kx + m\frac{d^2x}{dt^2}$$

$$\left[\text{same as when you start with } \sum F = ma\right]$$

Using Differential Equations to Solve for T for a Torsion Pendulum



$$\sum \tau = I\alpha$$

$$-\kappa\theta = I\alpha$$

$$0 = I\alpha + \kappa\theta$$

$$0 = I\frac{d^2\theta}{dt^2} + \kappa\theta$$

substituting $\frac{d^2\theta}{dt^2} = -\omega^2\theta_{max}\cos(\omega t + \phi)$ and $\theta = \theta_{max}\cos(\omega t + \phi)$:

$$0 = -I\omega^2\theta_{max}\cos(\omega t + \phi) + \kappa\theta_{max}\cos(\omega t + \phi)$$

canceling $\theta_{max}\cos(\omega t + \phi)$ from both terms:

$$0 = -I\omega^2 + \kappa$$

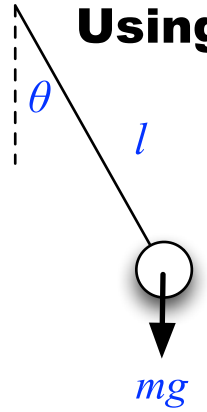
solving for ω:

$$\omega = \sqrt{\frac{\kappa}{I}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

Using Differential Equations to Solve for T for a Simple Pendulum



$\sum \tau = I\alpha$

$-mgl\sin\theta = I\alpha$

$0 = I\alpha + mgl\sin\theta$

$$0 = I\frac{d^2\theta}{dt^2} + mgl\sin\theta$$

for small angles, $\theta \approx \sin\theta$:

$$0 = I\frac{d^2\theta}{dt^2} + mgl\theta$$

subsituting: $\frac{d^2\theta}{dt^2} = -\omega^2\theta_{\max}\cos(\omega t + \phi)$ and $\theta = \theta_{\max}\cos(\omega t + \phi)$:

$0 = -I\omega^2\theta_{\max}\cos(\omega t + \phi) + mgl\theta_{\max}\cos(\omega t + \phi)$

canceling: $\theta_{\max}\cos(\omega t + \phi)$ from both terms:

$0 = -I\omega^2 + mgl$

for a small object, far from the pivot: $I = ml^2$

$0 = -ml^2\omega^2 + mgl$

canceling ml from both terms:

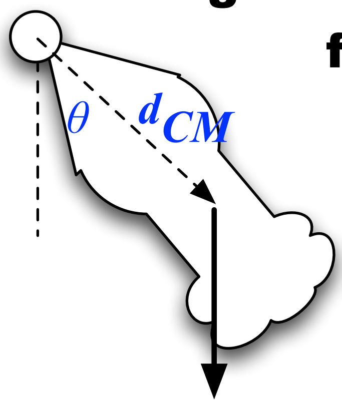
$0 = -l\omega^2 + g$

$$\omega = \sqrt{\frac{g}{l}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Using Differential Equations to Solve for T for a Physical Pendulum



$$\sum \tau = I\alpha$$

$$-mgd_{cm}\sin\theta = I\alpha$$

$$0 = I\alpha + mgd_{cm}\sin\theta$$

$$0 = I\frac{d^2\theta}{dt^2} + mgd_{cm}\sin\theta$$

for small angles, $\theta \approx \sin\theta$:

$$0 = I\frac{d^2\theta}{dt^2} + mgd_{cm}\theta$$

subsituting: $\frac{d^2\theta}{dt^2} = -\omega^2\theta_{\max}\cos(\omega t + \phi)$ and $\theta = \theta_{\max}\cos(\omega t + \phi)$:

$$0 = -I\omega^2\theta_{\max}\cos(\omega t + \phi) + mgd_{cm}\theta_{\max}\cos(\omega t + \phi)$$

canceling: $\theta_{\max}\cos(\omega t + \phi)$ from both terms:

$$0 = -I\omega^2 + mgd_{cm}$$

$$\omega = \sqrt{\frac{mgd_{cm}}{I}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{mgd_{cm}}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mgd_{cm}}}$$