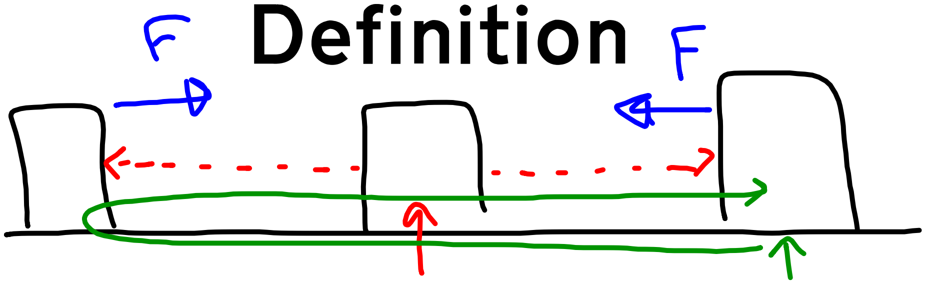
# Simple Harmonic Motion

Definition of SHM
Modeling it
Position, Velocity & Acceleration
Energy
Examples

# Definition




1. There is an EQUILIBRIUM POINT where the force is zero.

2. There is a RESTORING FORCE that is proportional to displacement from the equilibrium pt, but opposite in direction.

$$F \propto -x$$

# Modeling It

**One complete back and forth:** oscillation, cycle, or vibration

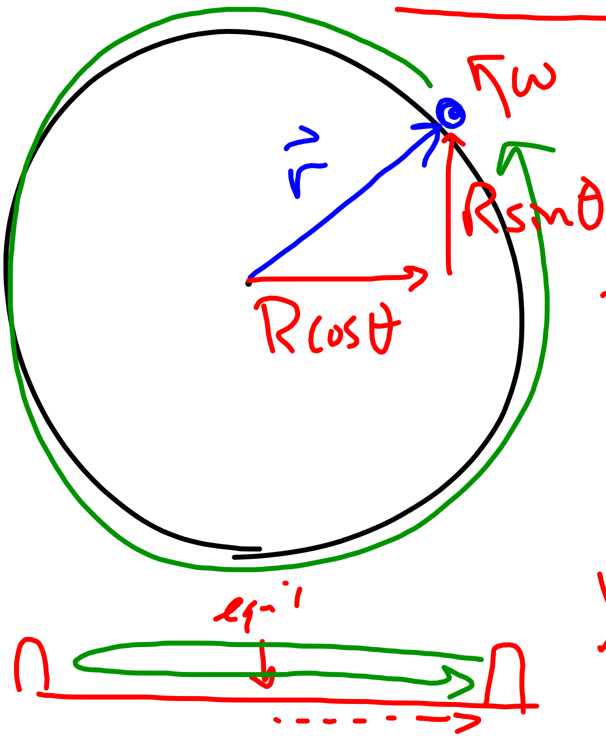
**number of oscillations/sec:** Frequency in Hertz (Hz) $f$

**number of seconds for one oscillation:** Period (s) $T$

$f \quad \frac{osc}{sec} \qquad \frac{sec}{osc} \quad T$

$T = \frac{1}{f}$

$$\vec{r}(t) = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$




$\theta$ - changing

$$\vec{r}(t) = R\cos(\omega t)\,\hat{i} + R\sin(\omega t)\,\hat{j}$$

rad/s

$$X(t) = X_{max}\cos(\omega t)$$

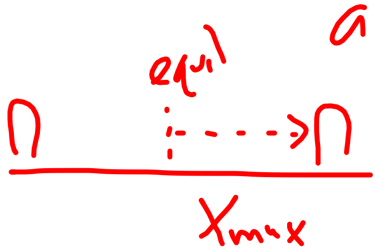
## Trig Functions > pick cosine b/c starts at +max

$$x(t) = x_{\max} \cos\overbrace{(\omega t + \phi)}^{\text{phase angle}}$$

amplitude

angular frequency

phase constant




$\omega = 2\pi f$

What information can you glean about this SH Oscillator from its position function?

(x is in meters when t is in seconds.)

$$x(t) = 5\cos(8\pi t)$$

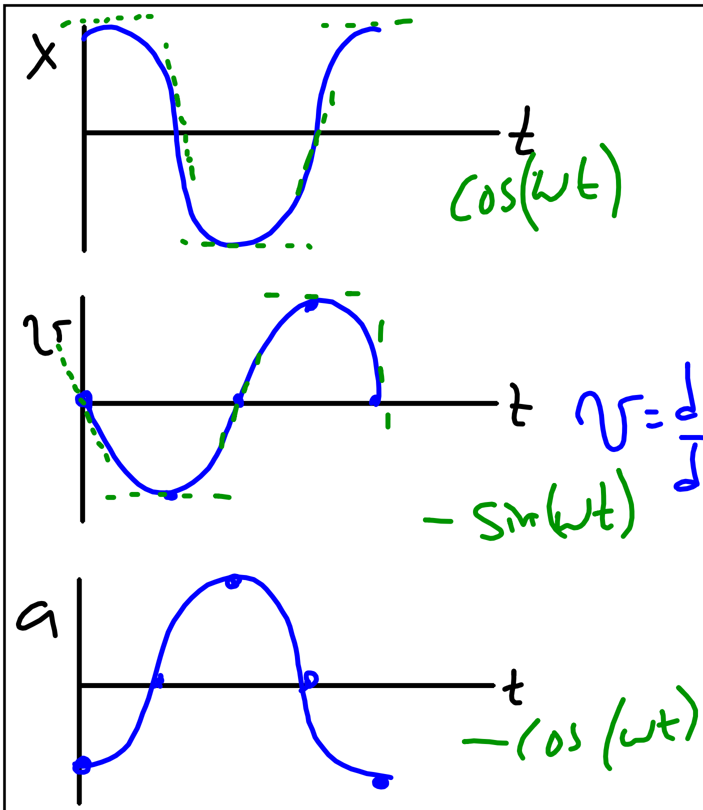
SHO $= x_{max}\cos(\omega t + \phi)$

$x_{max} = 5\,m$

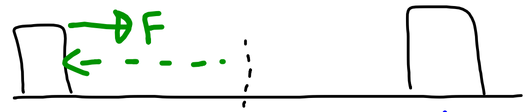
$\omega = 8\pi \frac{rad}{s}$

$\phi = 0$

# Three graphs




$$v = \frac{dx}{dt}$$




| | | |
|---|---|---|
| $-X_{min}$ | equil | $+X_{max}$ |
| $v=0$ | $\pm v_{max}$ | $v=0$ |
| $+a_{max}$ | $a=0$ | $-a_{max}$ |

$$x = 5\cos(12\pi t)$$

$$v = -(12\pi)5\sin(12\pi t) = -60\pi\sin(12\pi t)$$

$$a = -60\pi(12\pi)\cos(12\pi t) = -720\pi^2\cos(12\pi t)$$

constants

$$x = x_{max} \cos(\omega t + \phi)$$

$$v = -\overbrace{\omega x_{max}}^{v_{max}} \sin(\omega t + \phi)$$

$$a = -\overbrace{\omega^2 x_{max}}^{a_{max}} \cos(\omega t + \phi)$$

Calculate $v_{max}$ and $a_{max}$

$$x = \underbrace{5}_{x_{max}} \cos(\underbrace{12\pi}_{\omega} t)$$

$$v_{max} = \omega x_{max} = (12\pi)(5) = 188 \frac{m}{s}$$

$$a_{max} = \omega^2 x_{max} = (12\pi)^2(5) = 7{,}099 \frac{m}{s^2}$$

# position, velocity, acceleration

$$x(t) = x_{\max} \cos(\omega t + \phi)$$

$$v(t) = -\omega x_{\max} \sin(\omega t + \phi)$$

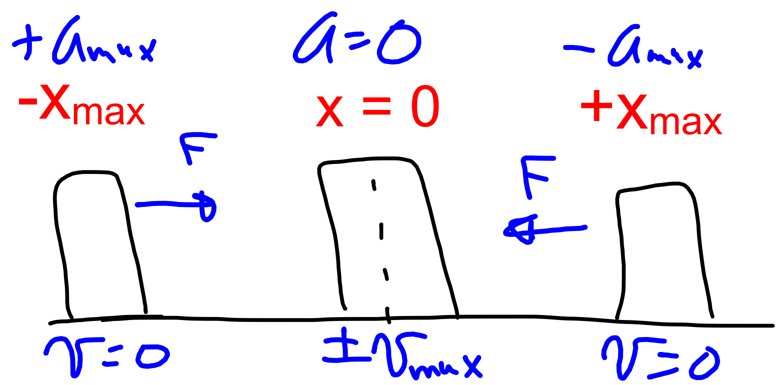
$$a(t) = -\omega^2 x_{\max} \cos(\omega t + \phi)$$

$$\omega = 2\pi f$$

$$T = \frac{1}{f}$$

$$a_{max} = \omega^2 x_{max}$$

$$v_{max} = \omega x_{max}$$




$$a = -\omega^2 x$$

**Ex 1** $x = (4m)\cos(6\pi t - \frac{\pi}{2})$

Determine ω $\boxed{\omega = 6\pi \frac{rad}{s}}$

Determine *T* $T = \frac{1}{f}$ $f = \frac{\omega}{2\pi}$ $\frac{6\pi}{2\pi} = 3Hz$

$\boxed{T = 0.33 sec}$

*Find the phase angle at t = 3 s*

Determine the phase constant

phase angle $= (6\pi t - \frac{\pi}{2})$ $\boxed{\phi = -\frac{\pi}{2}}$

$= (6\pi(3) - \frac{\pi}{2})$

$= (18\pi - \frac{\pi}{2}) = \boxed{17\frac{1}{2}\pi}$

# Energy

$$E_{TOTAL} = U + K$$

$F \propto -x$

$F = -kx$

$U = \frac{1}{2}kx^2$

$$= \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$E = \frac{1}{2}k\left[X_{max}\cos(\omega t + \phi)\right]^2 + \frac{1}{2}m\left[-\omega X_{max}\sin(\omega t + \phi)\right]^2$$

at extremes: all energy is potential

$$E_{TOTAL} = \frac{1}{2}kX_{max}^2$$

at equil pt: all energy is kinetic

$$E_{TOTAL} = \frac{1}{2}mv_{max}^2$$

**Ex 2** **m= 0.5 kg**

$$x = (1m)\cos(4\pi t + \frac{\pi}{2})$$

Find v(t)

Find a(t)

Find a max and v max

Find F max

Find E total

$$v(t) = -4\pi \sin(4\pi t + \frac{\pi}{2})$$

$$a(t) = -16\pi^2 \cos(4\pi t + \frac{\pi}{2})$$

$$v_{max} = 4\pi \frac{m}{s} = 12.6 \frac{m}{s}$$

$$a_{max} = 16\pi^2 \frac{m}{s^2} = 158 \frac{m}{s^2}$$

$$F_{max} = m\, a_{max}$$

$$= (0.5\,kg)(158 \frac{m}{s^2}) = 79\,N$$

$$E_{TOTAL} = \frac{1}{2} m v_{max}^2 \leftarrow K \text{ at equil pt.}$$

$$= \frac{1}{2}(0.5\,kg)(12.6 \frac{m}{s})^2$$

$$= 39\,J$$

**Ex 3**

**phase constant = 0**
**$v_{max}$ = 4 m/s**
**$a_{max}$ = 8 m/s$^2$**
**m = 0.1 kg**

$$v_{max} = \omega x_{max}$$
$$a_{max} = \omega^2 x_{max}$$
$$a_{max} = \omega v_{max}$$

Find ω

$$8 = \omega 4$$
$$\boxed{2\,\frac{rad}{s} = \omega}$$

Find amplitude

write x(t), v(t), and a(t)

$$v_{max} = \omega x_{max}$$
$$4\,\frac{m}{s} = \left(2\,\frac{rad}{s}\right) x_{max}$$
$$\boxed{2\,m = x_{max}}$$

$$x(t) = (2\,m)\cos(2t)$$
$$v(t) = -\left(4\,\frac{m}{s}\right)\sin(2t)$$
$$a(t) = -\left(8\,\frac{m}{s^2}\right)\cos(2t)$$