Rotational Dynamics

The Parallel Axis Theorem

Right Hand Rule: The real direction of CW and CCW rotation

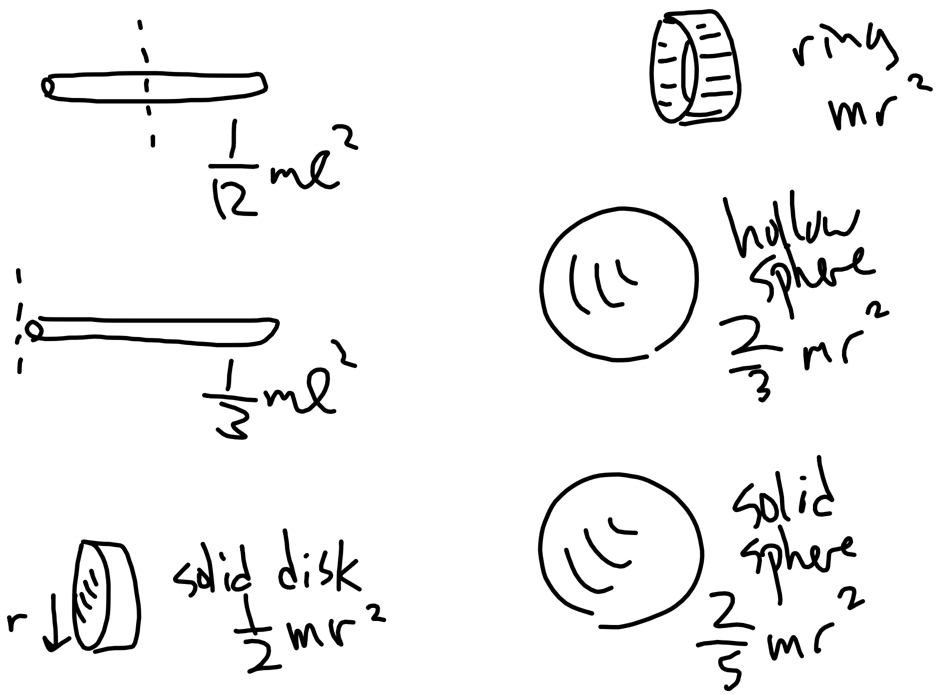
Dot and Cross Products

Angular Momentum

Conservation of Angular Momentum

Collisions with Fixed Pivots

Rotational Inertia list



Parallel Axis Theorem

Calculates the new rotational inertia when you move the pivot away from the CM

$$I_{NEW} = I_{CM} + md^2$$

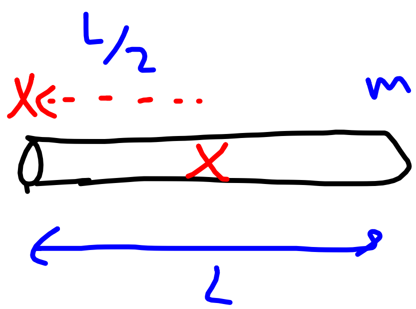
Old rotational inertia around center of mass

distance the pivot was shifted

mass of object

$$I_{NEW} = I_{CM} + md^2$$

Use the Parallel Axis Theorem to show that the rotational inertia of a rod around its end is $\frac{1}{3}mL^2$



$$\frac{1}{12}mL^2$$

$$I_{new} = I_{cm} + md^2$$

$$= \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12}mL^2 + \frac{1}{4}mL^2$$

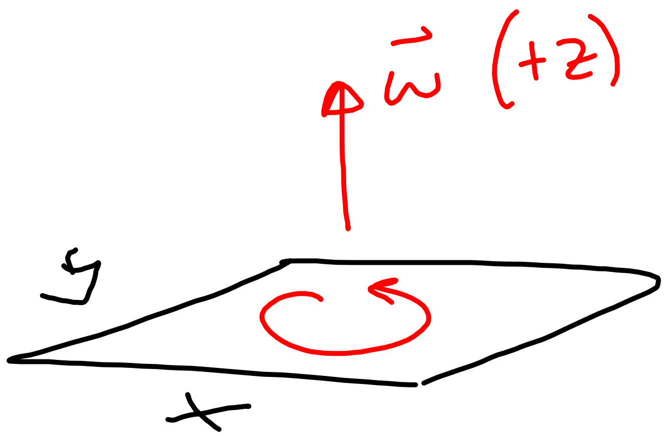
$$= \frac{1}{12}mL^2 + \frac{3}{12}mL^2$$

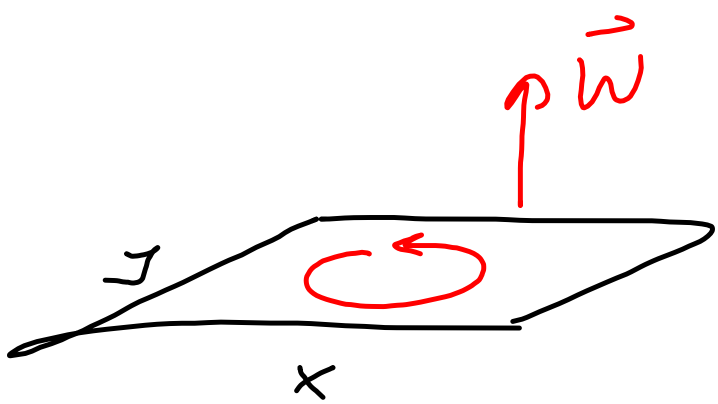
$$= \frac{4}{12}mL^2 = \frac{1}{3}mL^2 \checkmark$$

The Real Direction of CW and CCW

The Right Hand Rule

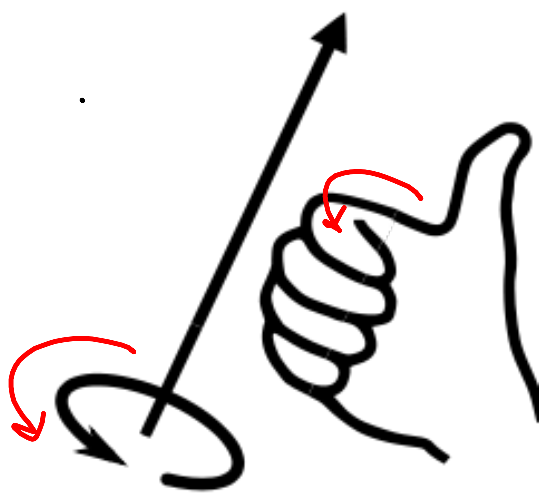
$\vec{\omega}$





1st Right Hand Rule: Angular Velocity

Stick your right thumb out. Curl the fingers of your right hand with the rotation; your thumb points in the direction of the angular velocity.



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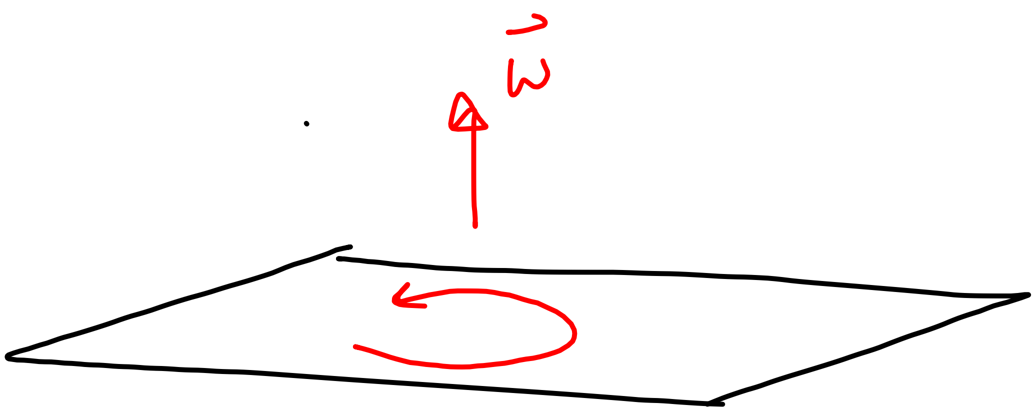


Two Ways to Multiply Vectors

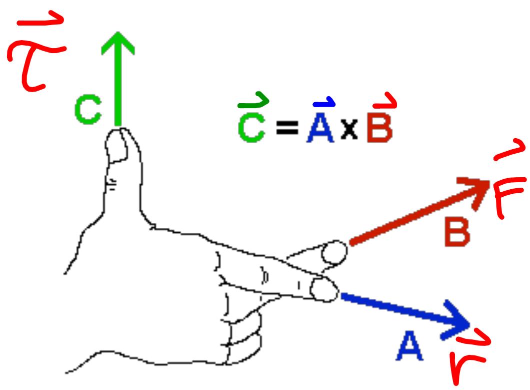
$\vec{A}$ $\vec{B}$

Cross Product	Dot Product
$\vec{A} \times \vec{B}$	$\vec{A} \cdot \vec{B}$
$AB\sin\theta$	$AB\cos\theta$
When perpendicularity matters	When parallel-ness matters
$\tau = rF\sin\theta$	$W = Fs\cos\theta$
$\vec{\tau} = \vec{r} \times \vec{F}$	$W = \vec{F} \cdot \vec{s}$
vector	scalar

2nd Right Hand Rule: Cross Product

Orient the index finger, middle finger and thumb of your right hand to be at 90 degrees to each other.

- The index finger points with the 1st vector in the product.
- The middle finger points with the 2nd vector in the product.
- Your thumb will point in the direction of the cross product answer.

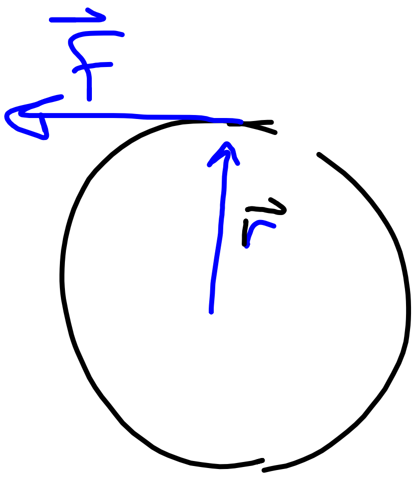


$\vec{\tau} = \vec{r} \times \vec{F}$

2nd Right Hand Rule: Cross Product

Move only your wrist (not your fingers) so that your:

- index finger points with the 1st vector in the product.
- middle finger points with the 2nd vector in the product.
- your thumb will point in the direction of the cross product answer.



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$\vec{\tau}$ out of screen

$\odot$ $+z$

How do you draw a vector coming out of the page or into it?

$\odot$

Out of page or screen

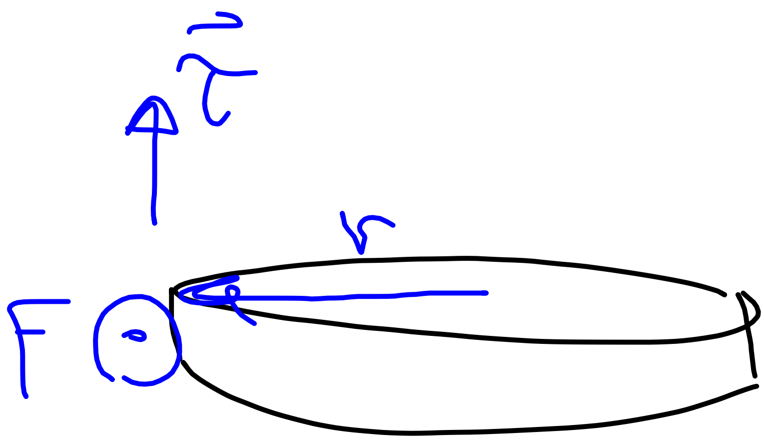
point

$\otimes$

Into page or screen

tail feathers

Use the right-hand rule to find the direction of torque.



Find the dot product and the cross product of the vectors below.

$$\vec{A} = 3\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{B} = 2\hat{i} - 6\hat{j} + 1\hat{k}$$

$$\vec{A} \cdot \vec{B} = (3)(2) + (4)(-6) + (-5)(1)$$

$$= 6 - 24 - 5$$

$$= -23$$

Find the dot product and the cross product of the vectors below.

$$\vec{A} = 3\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{B} = 2\hat{i} - 6\hat{j} + 1\hat{k}$$

$$\vec{A} \times \vec{B}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & -5 \\ 2 & -6 & 1 \end{vmatrix} = \begin{matrix} \hat{i}\left((4)(1) - (-5)(-6)\right) \\ -\hat{j}\left((3)(1) - (2)(-5)\right) \\ \hat{k}\left((3)(-6) - (2)(4)\right) \end{matrix}$$

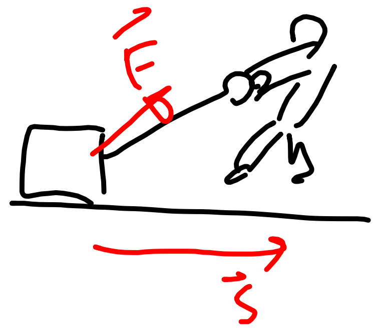
$$(4-30)\hat{i} - (3+10)\hat{j} + (-18-8)\hat{k}$$

$$-26\hat{i} - 13\hat{j} - 26\hat{k}$$

Find the work done by the force over the displacement given.

$$\vec{F} = 3\,\hat{i} + 2\,\hat{j}$$

$$\vec{s} = 6\,\hat{i} + 0\,\hat{j}$$



$$= (3)(6) + (2)(0) \qquad W = \vec{F} \cdot \vec{s}$$

$$= 18 + 0$$

$$W = 18\ \text{J}$$ scalar

Find the torque for the force and radius vectors below.

$$\vec{r} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) m$$

$$\vec{F} = \left(-1\hat{i} + 2\hat{j} - 3\hat{k}\right) N$$

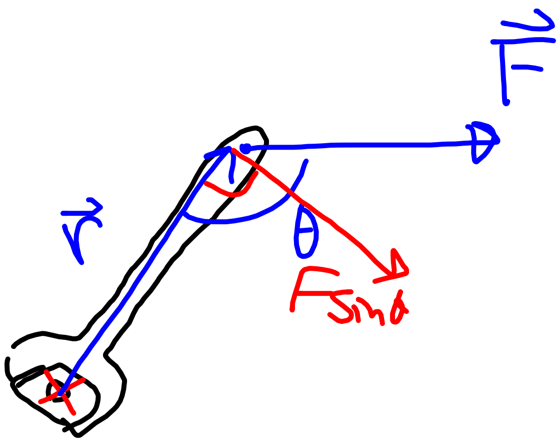
$\vec{r} \times \vec{F}$

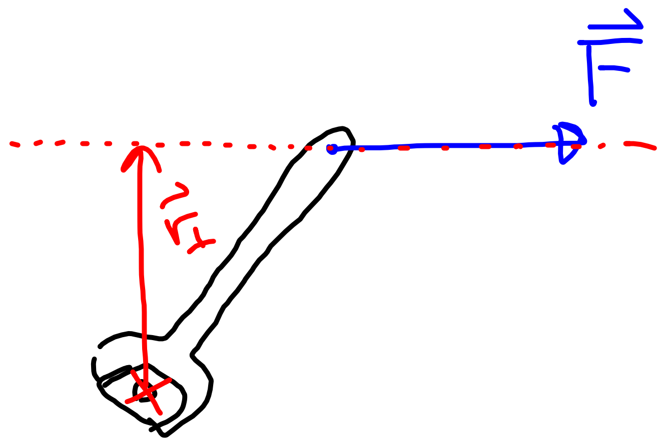
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 2 & -3 \end{vmatrix} = \begin{matrix} \hat{i}\left((3)(-3) - (2)(5)\right) \\ -\hat{j}\left((2)(-3) - (-1)(5)\right) \\ \hat{k}\left((2)(2) - (-1)(3)\right) \end{matrix}$$

$$(-9-10)\hat{i} - (-6+5)\hat{j} + (4+3)\hat{k}$$

$$\left(-19\hat{i} + 1\hat{j} + 7\hat{k}\right) mN$$

Using r-perpendicular to Calculate Torque





$$\tau = rF\sin\theta$$

$$\tau = r_\perp F$$

Linear vs Angular Momentum

$$\vec{p} = m\vec{v}$$

$kg\left(\frac{m}{s}\right)$

$$\vec{L} = I\vec{\omega}$$

rotating objects

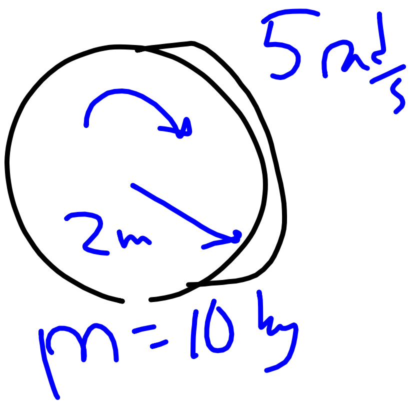
$$\vec{L} = \vec{r} \times \vec{p}$$

linearly moving objects

$kgm^2\left(\frac{rad}{s}\right)$

$kg\frac{m^2}{s}$

Find the angular momentum of a disk, mass 10 kg, radius 2 meters, rotating clockwise about its CM at 5 rad/s.



$$\vec{L} = I\vec{\omega}$$

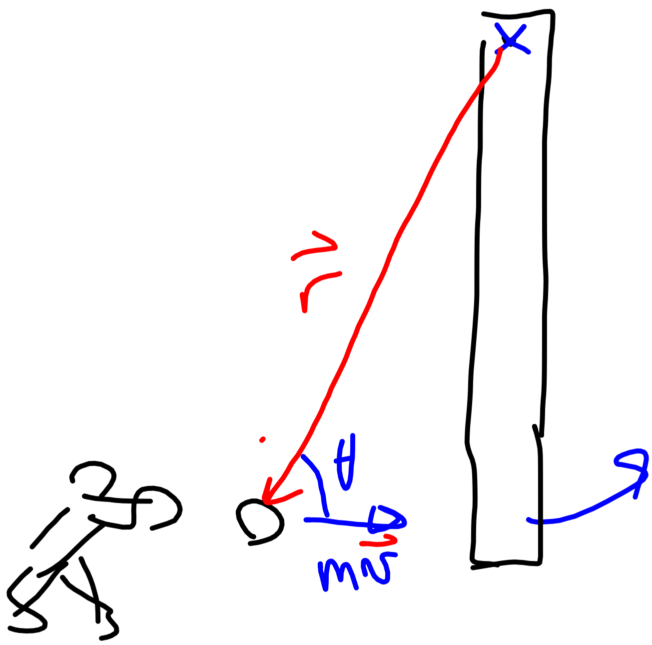
$$L = I\omega$$

$$= \left(\tfrac{1}{2}mr^2\right)(\omega)$$

$$= \tfrac{1}{2}(10)(2)^2(5)$$

$$= -100\ \frac{\mathrm{kg\,m^2}}{\mathrm{s}}\ \hat{k}$$

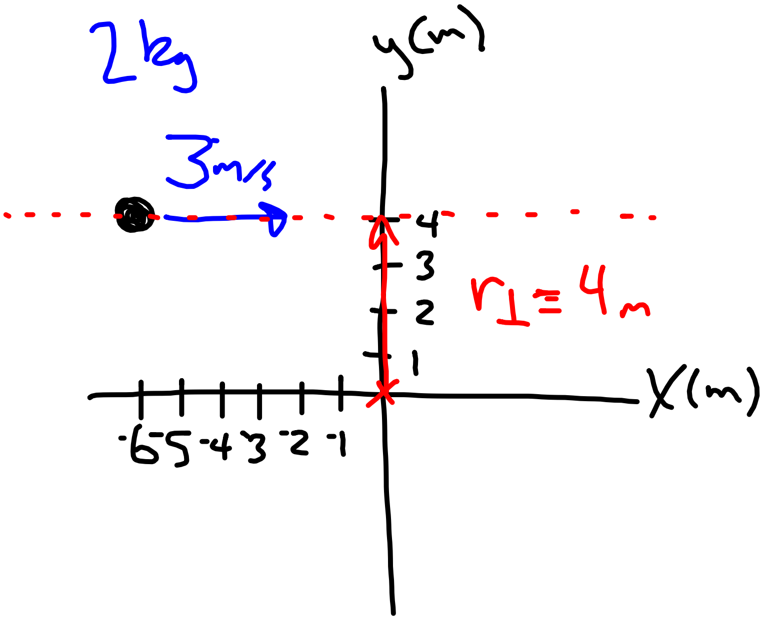
Does a LINEARLY moving object have ANGULAR momentum?



$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = r\,m v \sin\theta$$

The ball moves with the constant velocity shown. Find its angular momentum. Its current position: (-6, 4) m.



$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = r_{\perp} p$$

$$= (4m)(2kg)\left(3\frac{m}{s}\right)$$

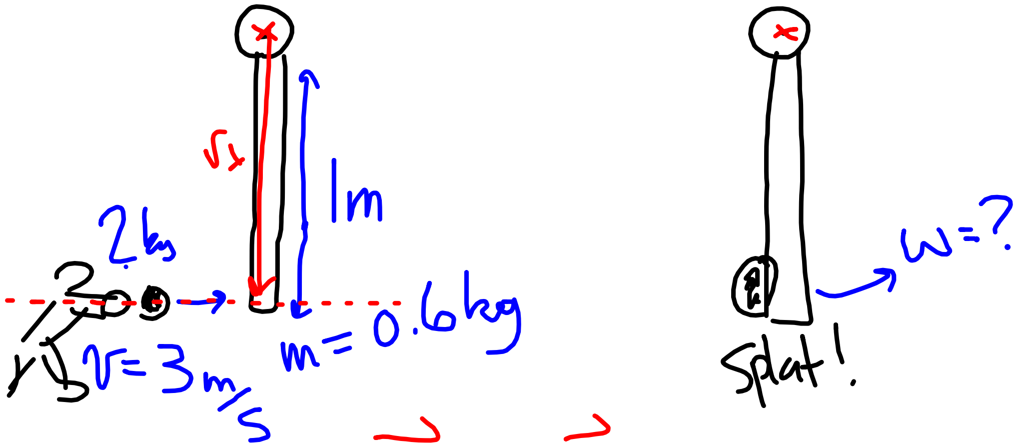
$$= {}^{-}24\ kg\frac{m^2}{s}\ \hat{k}$$

Law of Conservation of Angular Momentum

If there are no outside torques, angular momentum is conserved.

$$\vec{L}_i = \vec{L}_f$$

Find the angular velocity after the totally inelastic collision.
The bar has a fixed pivot at the top.
(subquestion: why can't you use linear momentum?)



$$\vec{L}_i = \vec{L}_f$$

$$r_\perp p = I\omega$$

$$(1\,\text{m})(2\,\text{kg})(3\,\tfrac{\text{m}}{\text{s}}) = (I_{bar} + I_{ball})\,\omega$$

$$6\,\frac{\text{kg m}^2}{\text{s}} = \left(\tfrac{1}{3}(0.6\,\text{kg})(1)^2 + (2\,\text{kg})(1\,\text{m})^2\right)\omega$$

$$6\,\frac{\text{kg m}^2}{\text{s}} = (0.2\,\text{kg m}^2 + 2\,\text{kg m}^2)\,\omega$$

$$6\,\frac{\text{kg m}^2}{\text{s}} = (2.2\,\text{kg m}^2)\,\omega$$

$$\boxed{2.72\,\frac{\text{rad}}{\text{s}} = \omega}$$

Force and Momentum; Torque and Angular Momentum

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\frac{d}{dt}(mv)$$

$$= m\frac{dv}{dt}$$

$$= ma$$

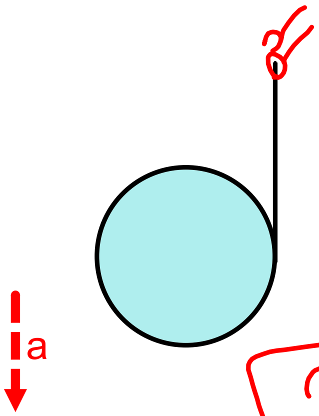
What is the Torque at t = 2 seconds?

$$\vec{L} = (5t^2)\hat{i} + (2t+1)\hat{j} + (4)\hat{k}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = (10t)\hat{i} + (2)\hat{j}$$

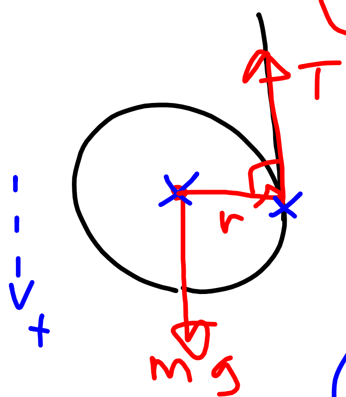
$$\vec{\tau}(2) = \left(20\hat{i} + 2\hat{j}\right) \text{ mN}$$

Torque and Rolling



The yo-yo is unwinding and falling while the top of the string is being held in place. The mass of the yo-yo is 0.02 kg. What is the acceleration of the yo-yo? Assume that the string unwinds from the edge and that the yo-yo is approximately a disk.

$$\boxed{v = r\omega \quad a = r\alpha}$$



$$\sum F = ma$$

$$mg - T = ma$$

$$(0.02)(10) - T = 0.02a$$

$$0.2 - T = 0.02a$$

$$0.2 - (0.01a) = 0.02a$$

$$0.2 = 0.03a$$

$$6.67\,\frac{m}{s^2} = a$$

$$\sum \tau = I\alpha$$

$$rT = \frac{1}{2}mr^2\alpha$$

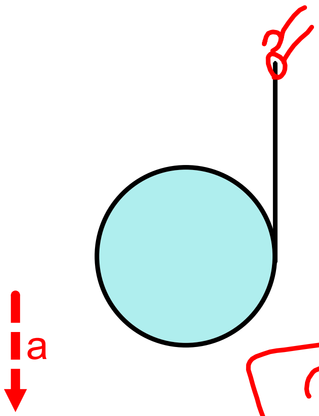
$$T = \frac{1}{2}mr\alpha$$

$$T = \frac{1}{2}mr\left(\frac{a}{r}\right)$$

$$T = \frac{1}{2}ma$$

$$T = 0.01a$$

Torque and Rolling



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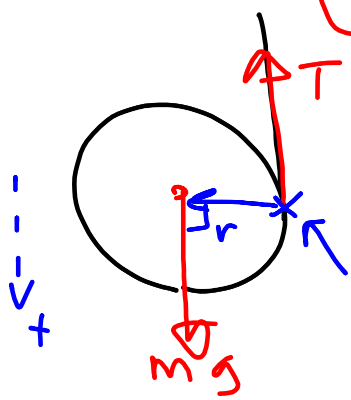

$$\sum \tau = I\alpha$$

$$rmg = \left(\frac{3}{2}mr^2\right)\left(\frac{a}{r}\right)$$

$$I_{new} = I_{cm} + md^2$$

$$= \frac{1}{2}mr^2 + mr^2$$

$$= \frac{3}{2}mr^2$$

$$mg = \frac{3}{2}ma$$

$$\frac{2}{3}g = a$$

$$T = 0.01a$$