Proof that Huygen's Relation is Equivalent to Conservation of Momentum and Kinetic Energy

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

(1) Conservation of Momentum.

$$m_1 v_{1i} - m_1 v_{1f} = m_2 v_{2f} - m_2 v_{2i}$$

(2) Re-arranging (1) so that mass 1 and mass 2 terms are on separate sides.

$$m_1 \left(v_{1i} - v_{1f} \right) = m_2 \left(v_{2f} - v_{2i} \right)$$

(3) Factoring out m's.

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

(4) Conservation of Kinetic Energy in an elastic collision.

$$\tfrac{1}{2} m_1 v_{1i}^2 - \tfrac{1}{2} m_1 v_{1f}^2 = \tfrac{1}{2} m_2 v_{2f}^2 - \tfrac{1}{2} m_2 v_{2i}^2$$

(5) Re-arranging (4) so that mass 1 and mass 2 terms are on separate sides.

$$m_1 \left(v_{1i}^2 - v_{1f}^2 \right) = m_2 \left(v_{2f}^2 - v_{2i}^2 \right)$$

(6) Factoring out m's and canceling out one-halfs from (5).

$$m_1 \left(v_{1i} - v_{1f} \right) \left(v_{1i} + v_{1f} \right) = m_2 \left(v_{2f} - v_{2i} \right) \left(v_{2f} + v_{2i} \right)$$

(7) Factoring the difference between two squares in (6) (remember that?)

$$\frac{\cancel{m_1} \cancel{\left(v_{1i} - v_{1f} \right)} \left(v_{1i} + v_{1f} \right) = \cancel{m_2} \cancel{\left(v_{2f} - v_{2i} \right)} \left(v_{2f} + v_{2i} \right)}{\cancel{m_1} \cancel{\left(v_{1i} - v_{1f} \right)} = \cancel{m_2} \cancel{\left(v_{2f} - v_{2i} \right)}}$$

(8) Dividing (7) by (3) and canceling.

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

(9) Huygen's Relation is left, quod erat demonstratum.