SYSTEMS OF PARTICLES AND MOMENTUM

1. Center of mass: particles and objects
2. Another view of systems
3. Isolated Systems
4. From Newton's 2nd Law to Impulse & Change in momentum
5. Impulse-Momentum vs Work-Kinetic Energy

SYSTEMS OF PARTICLES AND CM

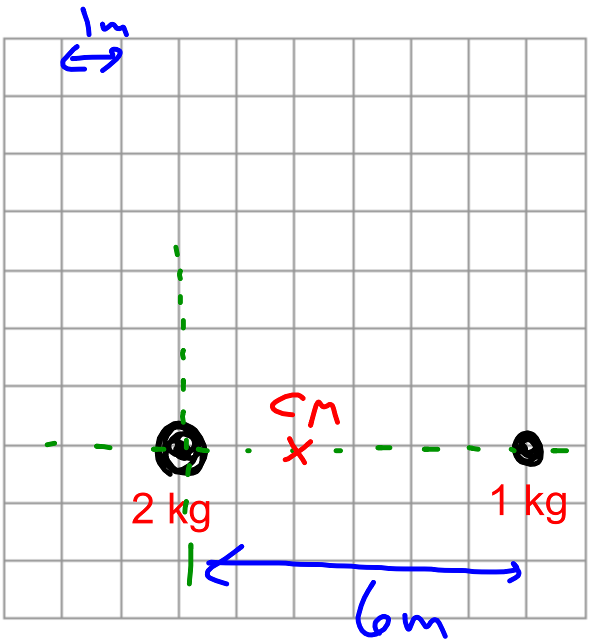
Center of mass = the average position of all the pieces of mass in the system

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{M_{total}}$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + \ldots}{M_{total}}$$

1-D CM

Find the center of mass of the system



(each block is 1 m x 1 m)

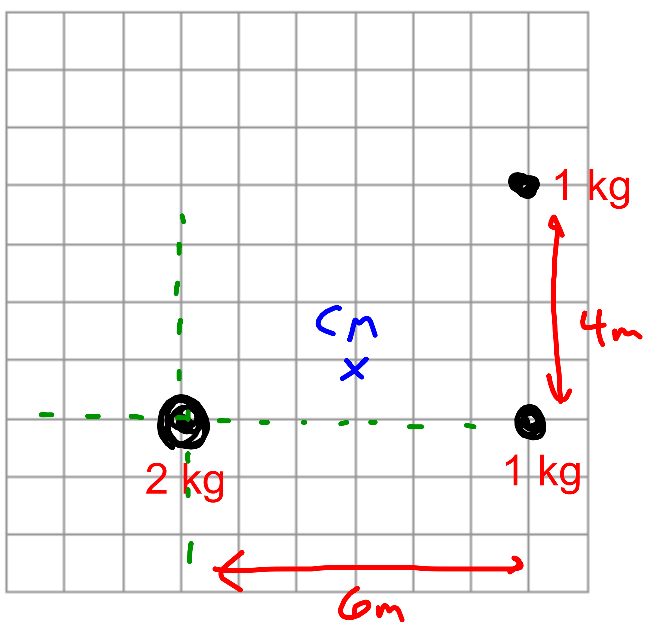
$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{M_{total}}$$

$$= \frac{(2\,kg)(0) + (1\,kg)(6m)}{3\,kg}$$

$$= \frac{6\,kg\,m}{3\,kg} = \boxed{2\,m}$$

2-D CM

Find the center of mass of the system



$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M_{total}}$$

$$= \frac{(2)(0) + (1)(6) + (1)(6)}{4\,kg}$$

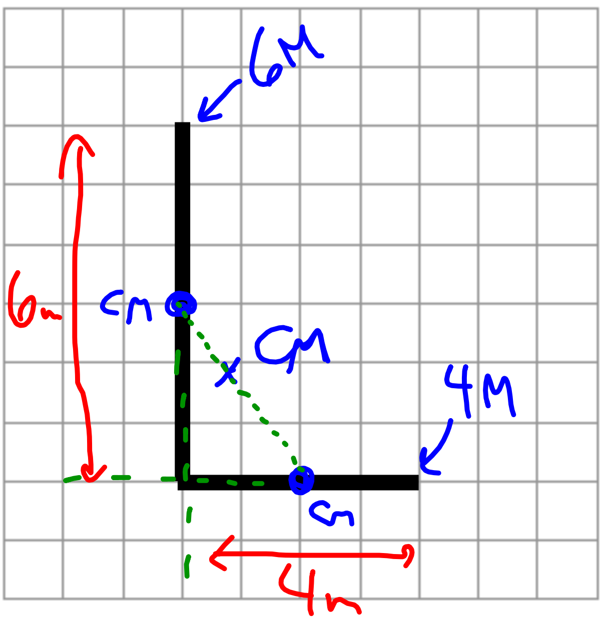
$$= \frac{12\,kgm}{4\,kg} = \boxed{3m}$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M_{total}}$$

$$= \frac{(2)(0) + (1)(0) + (1)(4)}{4\,kg} = \frac{4\,kgm}{4\,kg} = \boxed{1m}$$

2-D CM

Find the center of mass of the system



The "L" has uniform density

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{M_{total}}$$

let M = mass of a 1m segment

$$= \frac{(6M)(0) + (4M)(2)}{10M}$$

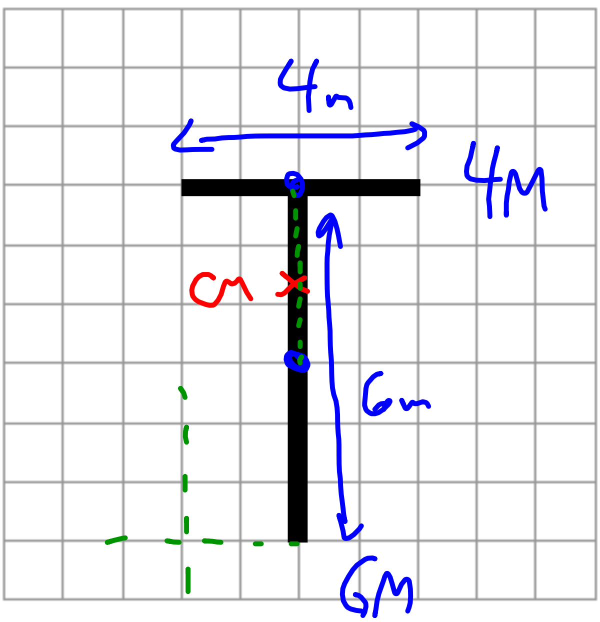
$$= \frac{8M}{10M} = \boxed{0.8m}$$

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2}{M_{total}}$$

$$= \frac{(6M)(3) + (4M)(0)}{10M} = \frac{18M}{10M} = \boxed{1.8m}$$

2-D CM

Find the center of mass of the system



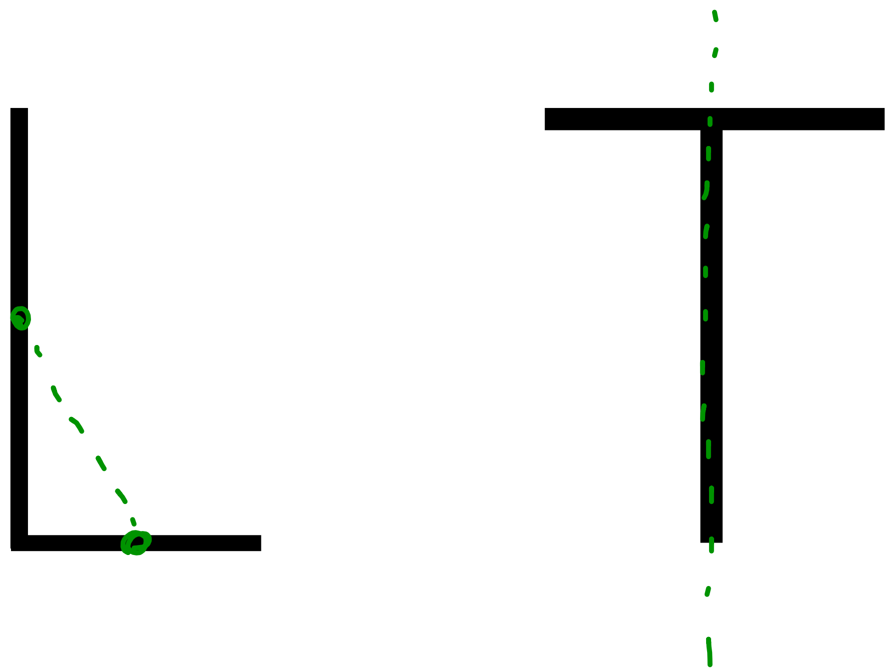
The "T" has uniform density

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{M_{total}} = \frac{(4M)(2) + (6M)(2)}{10M} = \frac{8M + 12M}{10M} = \frac{20M}{10M} = \boxed{2m}$$

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2}{M_{total}} = \frac{(6M)(3) + (4M)(6)}{10M}$$

$$= \frac{18M + 24M}{10M} = \frac{42M}{10M} = \boxed{4.2m}$$

Rules of Thumb & Symmetry Short Cuts



ANOTHER VIEW OF SYSTEMS

$$M_{total}X_{cm} = m_1x_1 + m_2x_2 + m_3x_3 + \ldots$$

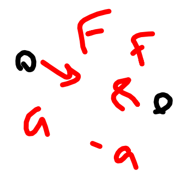
$$\frac{d}{dt}\left(M_{tot}X_{cm}\right) = \frac{d}{dt}\left(m_1x_1 + m_2x_2 + \ldots\right)$$

$$M_{tot}V_{cm} = m_1v_1 + m_2v_2 + \ldots$$

$$M_{tot}A_{cm} = m_1a_1 + m_2a_2 + \ldots$$

In an isolated system....

$$\sum F_{ext} = 0$$

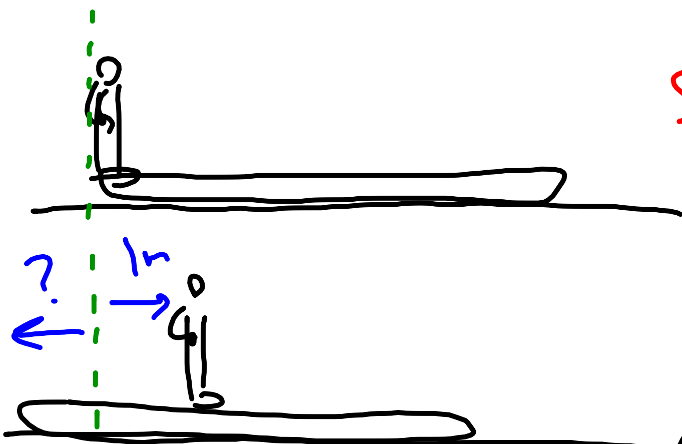


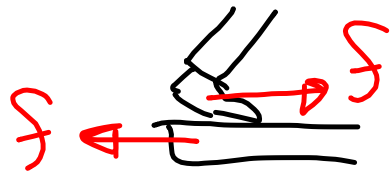
$$M_{total} A_{cm} = 0 \rightarrow M_{total} V_{cm} = \text{constant}$$

$$M_{total} V_{cm} = 0 \rightarrow M_{total} X_{cm} = \text{constant}$$

Isolated Systems

The 80 kg person is at the end of the 100 kg boat. If the person moves 1 m to the right (relative to the water), how far back did the boat move?





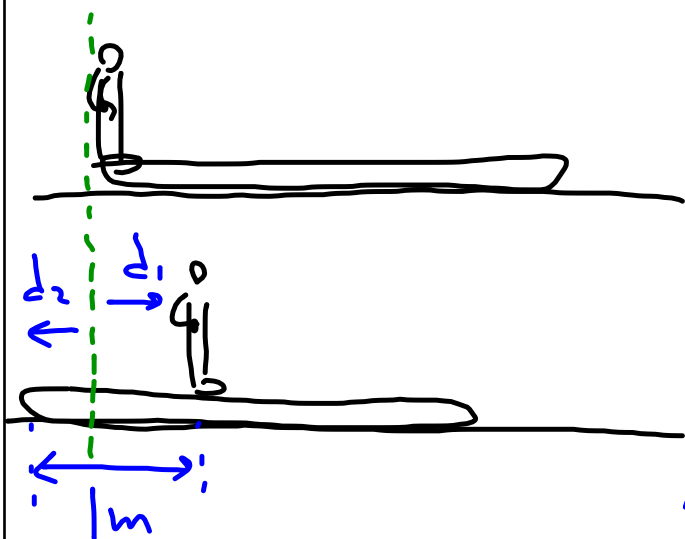
$$m_1 d_1 = m_2 d_2$$

$$(80)(1) = (100) d_2$$

$$0.8\,m = d_2$$

Isolated Systems

The 80 kg person is at the end of the 100 kg boat. If the person moves 1 m to the right (relative to the boat) how back did the boat move?



$$m_1 d_1 = m_2 d_2$$

$$80 d_1 = 100 d_2$$

$$d_1 + d_2 = 1_m$$

$$d_1 = 1 - d_2$$

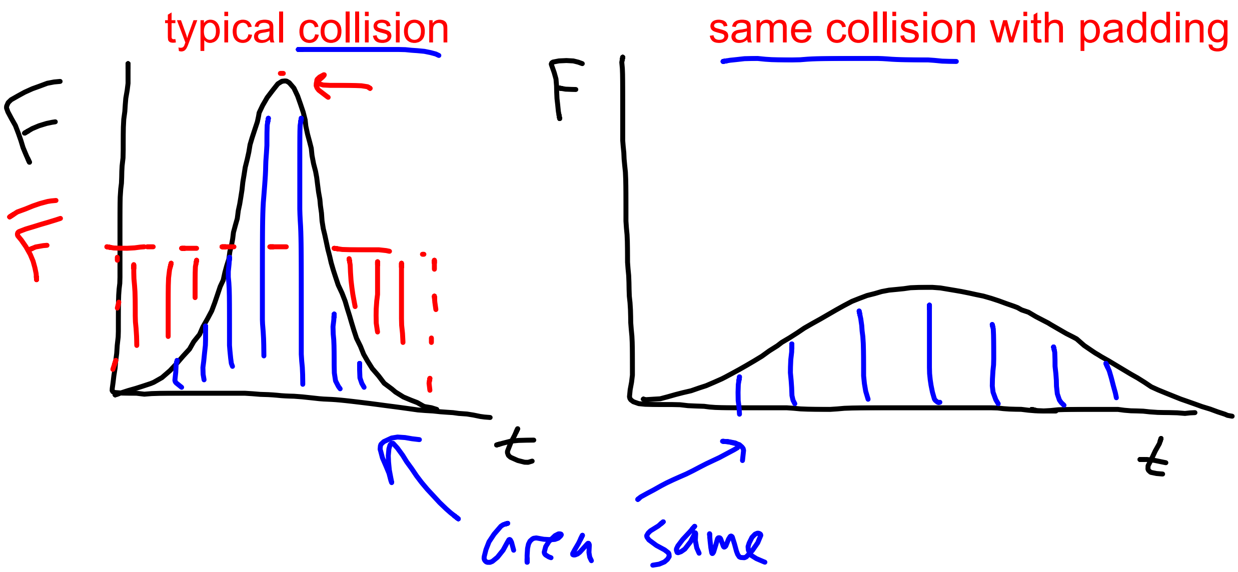
$$80(1 - d_2) = 100 d_2$$

$$80 - 80 d_2 = 100 d_2$$

$$80 = 180 d_2$$

$$0.44_m = d_2$$

The Problem with Collisions



$$J = \text{impulse} = \text{area on } F \text{ vs } t$$

$$J = \int F\,dt$$

Another look at Newton's 2nd Law

$$\sum F = ma$$

One force: $F = ma$

$$F = m\frac{dv}{dt}$$

$$\int F\,dt = \int m\,dv$$

impulse

$$= m\int_{v_i}^{v_f} dv$$

$$J = m(v_f - v_i)$$

momentum
$p = mv$

$$\boxed{J = \Delta p}$$

$$J = F\Delta t$$

Impulse and Change in Momentum

Impulse (in Ns) and change in momentum
(in kgm/s) calculate the same number

$$J = \Delta p$$

$\Delta p = m(v_f - v_i)$

$$J = \int F\,dt$$

$$J = \text{area under F vs t graph}$$

$$J = \bar{F}\Delta t$$

Calculating Impulse: Constant F

Find the impulse delivered by a constant 200 N force that acts for 0.1 seconds.

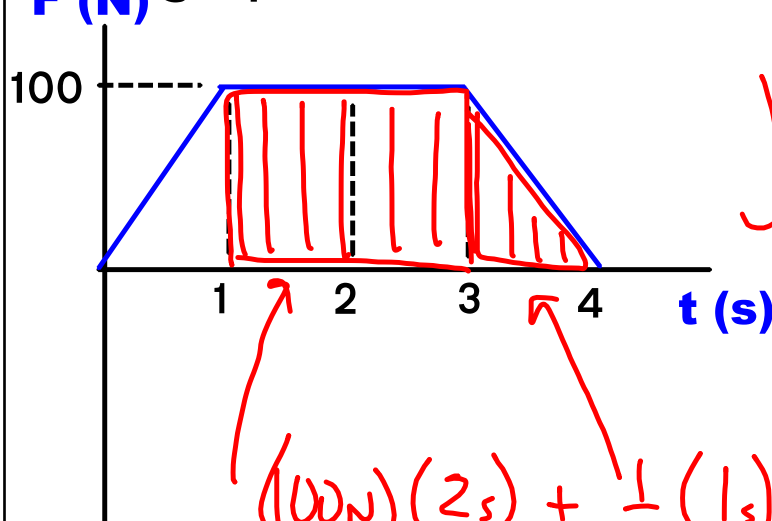
$J = F\Delta t = (200\,N)(0.1\,s)$

$= 20\,N\cdot s$

$\Delta p = 20\,kg\frac{m}{s}$

Calculating Impulse: F vs t graph

Find the impulse delivered by the force (graphed below) between t = 1 s and t = 4 s



$J = \text{area}$

$(100\,N)(2\,s) + \frac{1}{2}(1\,s)(100\,N)$

$J = 200\,Ns + 50\,Ns = 250\,N\cdot s$

Calculating Impulse: F as a function of t

Find the impulse delivered by the force F = (4t - 3) N from t = 1 s to t = 2 s

$$J = \int F\,dt = \int_1^2 (4t-3)\,dt$$

$$= \left[\frac{\cancel{4}^{2} t^2}{\cancel{2}} - 3t\right]_1^2$$

$$= \left[2(2)^2 - 3(2)\right] - \left[2(1)^2 - 3(1)\right]$$

$$(8-6) - (2-3)$$

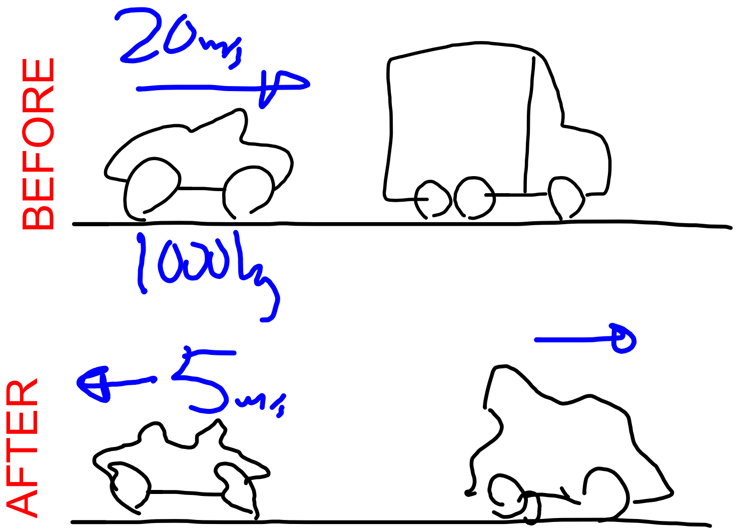
$$2 - (-1)$$

$$J = 3\ \mathrm{N\cdot s}$$

Calculating Impulse: momentum

a) Find the impulse delivered to the car and to the truck.

b) If the collision lasted 0.05 s, what was the average force on each one during the collision?



$$J_c = \Delta p_c$$
$$= m_c(v_f - v_i)$$
$$= (1000)(-5-20)$$
$$= (1000)(-25)$$
$$= -25000 \text{ kg m/s}$$

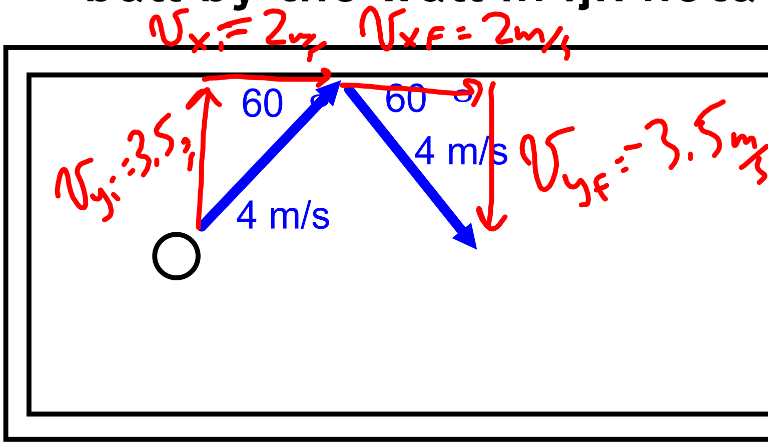
$$\therefore J_c = -25{,}000 \text{ N·s}$$
$$J_T = +25{,}000 \text{ N·s}$$

b) $J_c = \bar{F} \Delta t$

$$-25{,}000 \text{ N·s} = \bar{F}_c (0.05 \text{ s})$$
$$\boxed{-500{,}000 \text{ N} = \bar{F}_c}$$
$$\boxed{F_T = +500{,}000 \text{ N}}$$

Calculating Impulse: 2D

Find the impulse delivered to the 2 kg ball by the wall in ijk notation.



$J_x = \Delta p_x$

$J_y = \Delta p_y$

$$J_x = \Delta p_x = (2)(2-2) = 0$$

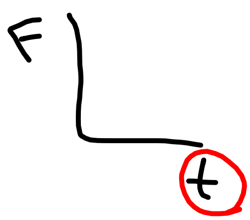
$$J_y = \Delta p_y = (2)(-3.5 - 3.5) = (2)(-7) = -14 \text{ N}\cdot\text{s}$$

$$\vec{J} = \left[(0)\hat{i} + (-14)\hat{j}\right] \text{ N}\cdot\text{s}$$

Side note:
Momentum-Impulse vs Work-Kinetic Energy

$$J = \Delta p$$

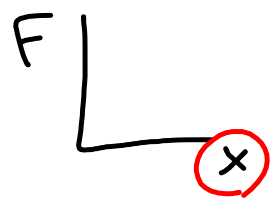
$$J = \int F\,dt$$



$$\Delta p = m(v_f - v_i)$$

$$W_{total} = \Delta K$$

$$W = \int F\,dx$$



$$\Delta K = \tfrac{1}{2} m\left(v_f^2 - v_i^2\right)$$