Work and Kinetic Energy

- Another Bridge between the World of Motion and the World of Forces
- The Work-Energy Theorem
- Work
- Power
- Kinetic Energy
- Springs
- Gravity
- Sample problems

Is there another way to bridge the worlds?

The World of Motion

position

velocity

accel

time

constant accel

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$x = x_0 + \tfrac{1}{2}(v + v_0) t$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

non-constant accel

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt}$$

$$x = \int v\,dt \qquad v = \int a\,dt$$

accel

The World of Forces

$$\sum F = ma$$

force

mass

accel

Forces-Kinematics Approach

The World of Motion

position

velocity

accel

time

constant accel

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$x = x_0 + \tfrac{1}{2}(v + v_0)t$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

non-constant accel

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt}$$

$$x = \int v\,dt \qquad v = \int a\,dt$$

$\longleftrightarrow$ accel $\longleftrightarrow$

The World of Forces

$$\boxed{\sum F = ma}$$

force

mass

accel

Work-Energy Approach

$$\boxed{\begin{aligned} W &= \vec{F}\cdot\vec{s} = Fs\cos\theta \\ W &= F_x\Delta x + F_y\Delta y + F_z\Delta z \\ W &= \int F\,ds \end{aligned}} \longleftarrow \boxed{W_{total} = \Delta K} \longrightarrow \boxed{\Delta K = \tfrac{1}{2}m(v_f^2 - v_i^2)}$$

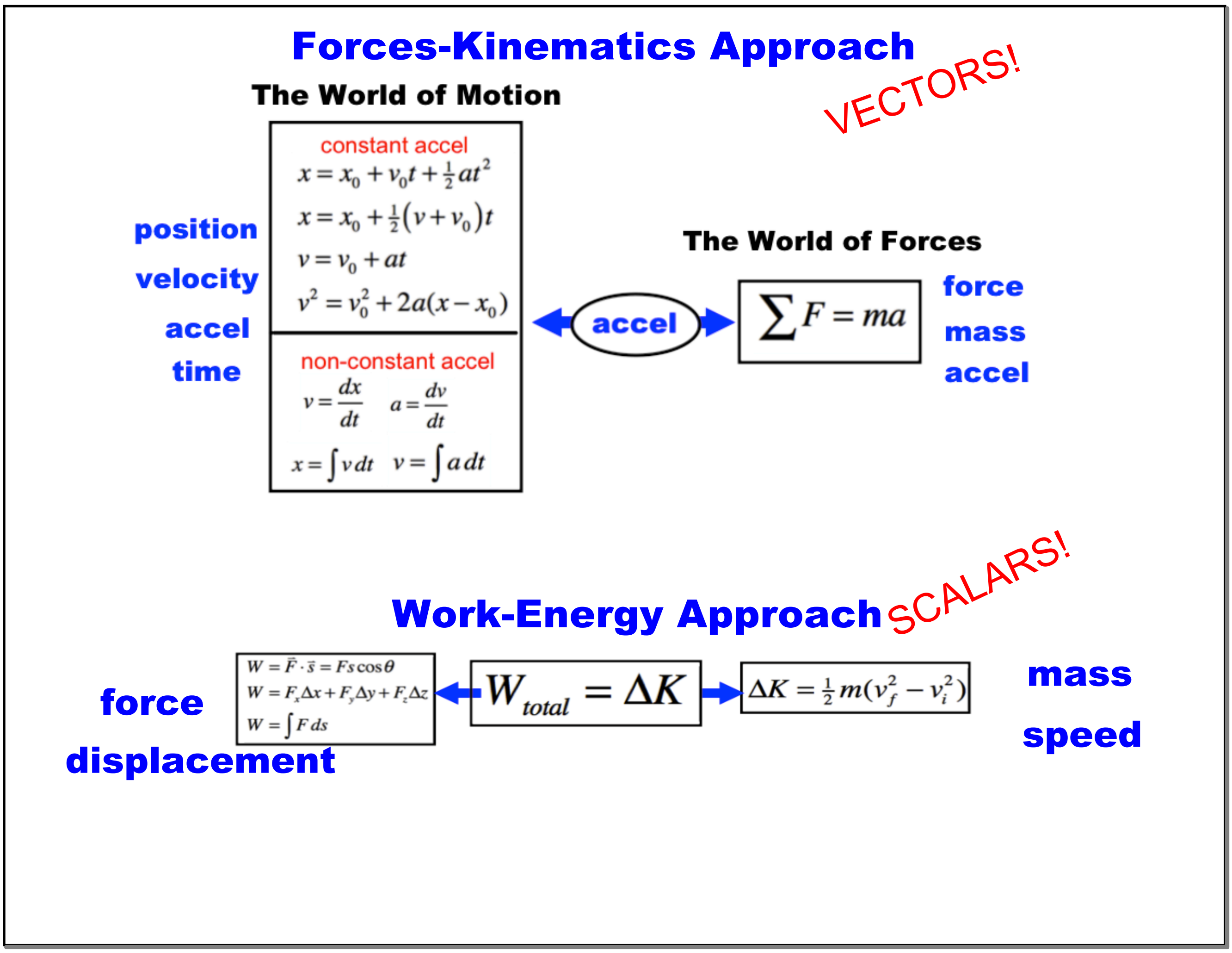
Work-Energy Theorem

$$\boxed{W_{total} = \tfrac{1}{2}m(v_f^2 - v_i^2)}$$

total work = change in kinetic energy

$W_{F_1} + W_{F_2} + W_{F_3} + \cdots$

$W_{net F}$


Forces-Kinematics Approach
VECTORS!
The World of Motion
position
velocity
accel
time
constant accel
$x = x_0 + v_0 t + \frac{1}{2}at^2$
$x = x_0 + \frac{1}{2}(v + v_0)t$
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non-constant accel
$v = \frac{dx}{dt}$ $a = \frac{dv}{dt}$
$x = \int v\,dt$ $v = \int a\,dt$
accel
The World of Forces
$\sum F = ma$
force
mass
accel
Work-Energy Approach
SCALARS!
force
displacement
$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$
$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$
$W = \int F\,ds$
$W_{total} = \Delta K$
$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$
mass
speed

Work

- is a scalar
- is measured in Nm, which is defined as a Joule (J)

$$W = \vec{F} \cdot \vec{s} = \underline{F}\,\underline{s}\cos\theta$$

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

$$W = \int F\,ds$$

$$= \int F_x\,dx + \int F_y\,dy + \int F_z\,dz$$

Power

- rate of doing work
- is measured in J/s which is a Watt

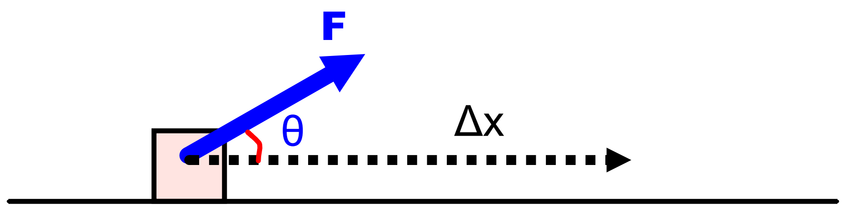
$$P_{avg} = \frac{\Delta W}{\Delta t}$$

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F} \cdot \vec{s}\right) = \vec{F} \cdot \vec{v}$$

(If Force is constant)

Pos & Neg Work

$$W = Fs\cos\theta$$

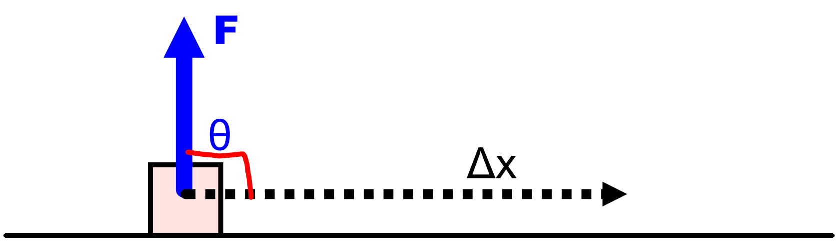


θ < 90°

Work is +

θ > 90°

Work is −



θ = 90°

Work is 0

Kinetic Energy

- is a scalar
- is measured in kgm^2/s^2 which is a Joule (J)

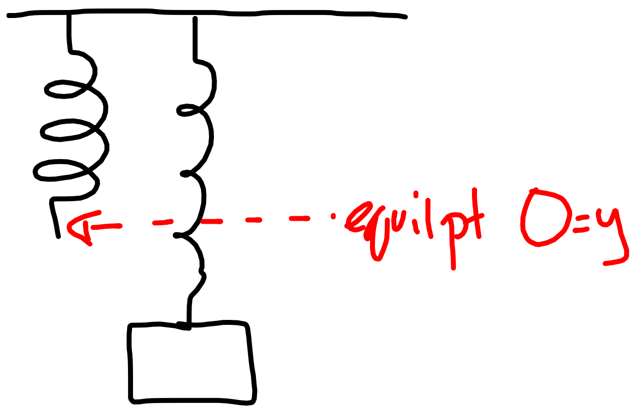
$$K = \frac{1}{2} m v^2$$

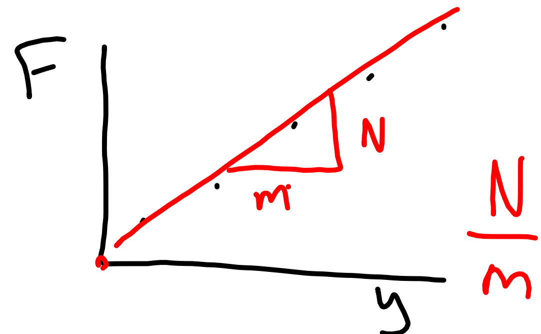
kg $\frac{m}{s}$

$kg\frac{m^2}{s^2} \rightarrow kg\frac{m}{s^2} \cdot m$

Spring Forces

- Have an equilibrium pt
- Have a restoring force
- That force is proportional to the displacement





$y = mx + b$

$F = ky$

$F = -ky$

$F = -kx$ (Hooke's Law)

Work done by a Spring

$$W_s = -\tfrac{1}{2}k\left(x_f^2 - x_i^2\right)$$

$$W = \int F\,ds$$

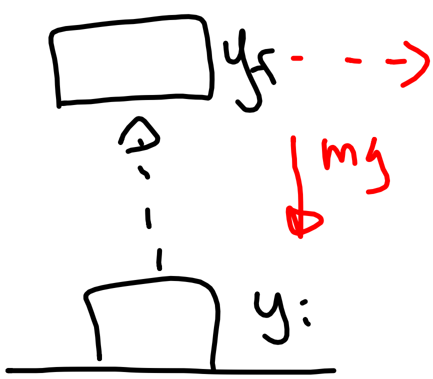
$$= \int F\,dx = \int_{x_i}^{x_f} (-kx)\,dx$$

$$= \left[-\frac{kx^2}{2}\right]_{x_i}^{x_f}$$

$$= -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

Work done by Gravity

$$W_g = -mg(y_f - y_i)$$



$$W = Fs\cos\theta$$

$$= (mg)(\Delta y)\cos 180^\circ$$

$$= -mg(y_f - y_i)$$

Work Energy Theorem

Relates the work done **ON** an object to the change in kinetic energy **OF** the object

$$W_{total} = \Delta K$$

example: Total work done on the truck is -100,000 J (by the engine, friction and drag). Find the truck's final v.

$$W_{total} = \Delta K$$

$$-100{,}000 = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$-100{,}000 = \frac{1}{2}(2000)(v_f^2 - 25^2)$$

$$-100{,}000 = 1000(v_f^2 - 625)$$

$$-100 = v_f^2 - 625$$

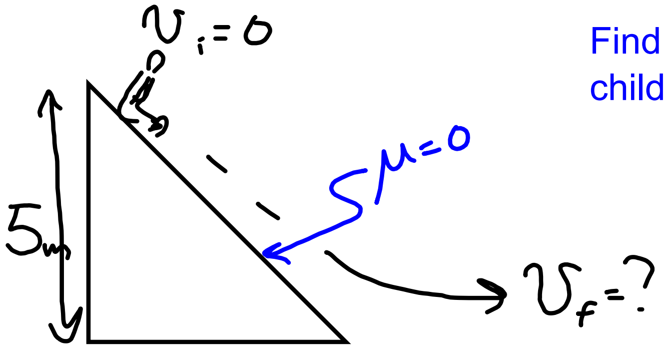
$$525 = v_f^2$$

$$\pm 22.9 \tfrac{m}{s} = v_f$$

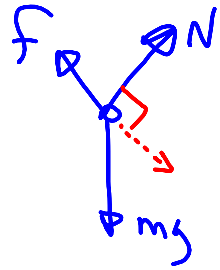
$$\pm 22.9 \tfrac{m}{s} = v_f$$

$$+22.9 \tfrac{m}{s} = v_f$$

ex: Another look at inclines



Find the final velocity of the child at the bottom of the slide.



$$W_{nnc} = \Delta K$$

$$W_f + W_N + W_g = \Delta K$$

$$0 + 0 - \cancel{m}g\,\Delta y = \frac{1}{2}\cancel{m}(v_f^2 - v_i^2)$$

$$-(10)(-5) = \frac{1}{2}(v_f^2 - 0)$$

$$50 = \frac{1}{2}v_f^2$$

$$100 = v_f^2 \qquad v_f = \pm 10\,\frac{m}{s}$$

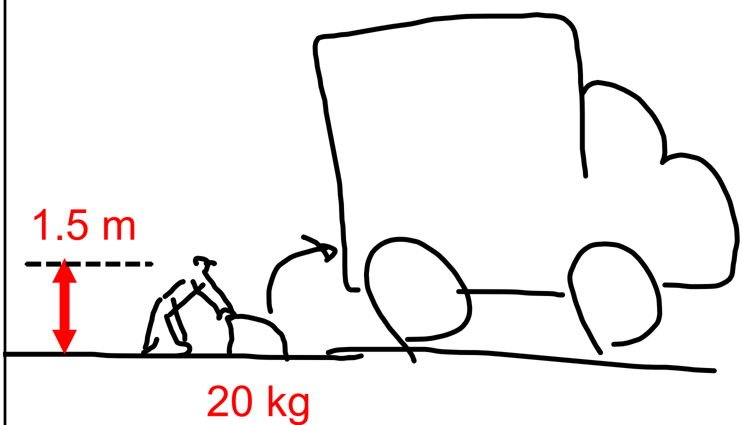
$$\boxed{v_f = +10\,\frac{m}{s}}$$

$$+100 = v_f^2$$

$$\pm 10\,\frac{m}{s} = v_f$$

$$\boxed{10\,\frac{m}{s} = v_f}$$

ex: Constant forces



The mover puts the 20 kg box in the back of the van. The box starts and ends at rest. Find the work done by gravity and the work done by the person.

$$W_p + W_g = \Delta K$$

$$W_p + W_g = 0$$

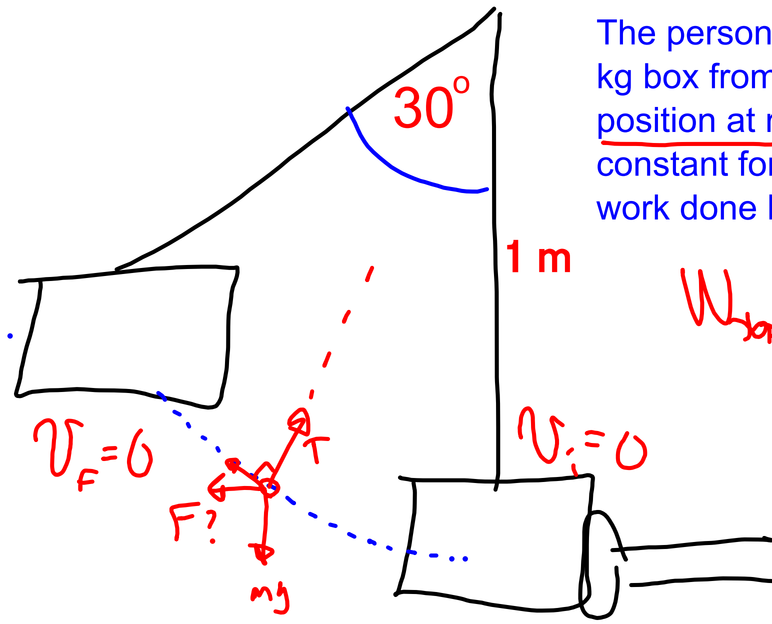
$$W_p - mg\Delta y = 0$$

$$W_p - (20)(10)(+1.5) = 0$$

$$W_p - 300 = 0$$

$$\boxed{W_p = +300\,J}$$

ex: Non-constant forces method #1



The person pushes the 10 kg box from rest to a higher position at rest with a non-constant force. Find the work done by the person.

$$W_{total} = \Delta K$$

$$W_p + W_T + W_g = \Delta K$$

$$W_p + 0 - mg\Delta y = 0$$

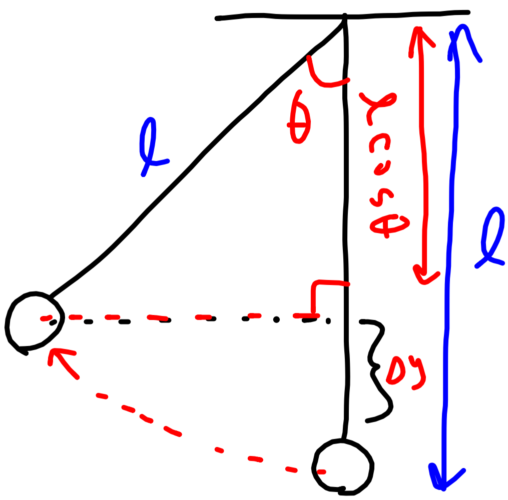
$$W_p - mg(l - l\cos\theta) = 0$$

$$W_p - (10)(10)(1 - \cos 30) = 0$$

$$W_p - 134 = 0$$

$$\boxed{W = +134\ J}$$

$$W_p = 134\ J$$



$$\Delta y = l - l\cos\theta$$

ex: Non-constant forces method #2

Find the work done by a force F = $3x^2$ from x = 1 to x = 2 m.

$$\cancel{W = F\Delta s}$$

$$W = \int F\,ds = \int_{x_i}^{x_f} F_x\,dx$$

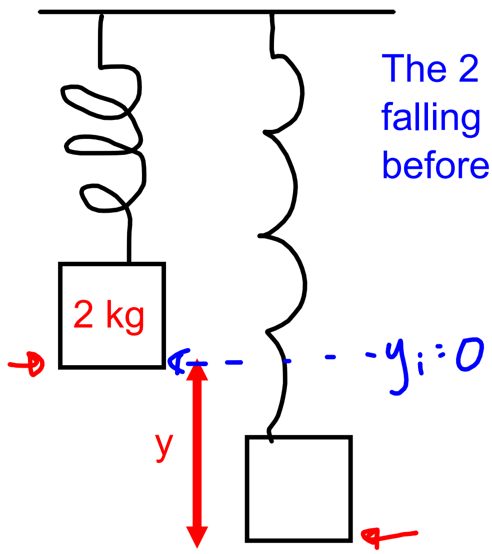
$$= \int_1^2 (3x^2)\,dx$$

$$= \left[\frac{\cancel{3}x^3}{\cancel{3}}\right]_1^2$$

$$= (2^3 - 1^3)$$

$$\boxed{= 7\ J}$$

ex: Spring vs gravity



The 2 kg mass is released from rest, falling y meters below where it started before coming to rest again. Find y.

$$W_{TOTAL} = \Delta K$$

$$W_s + W_g = 0$$

k = 200 N/m

$$-\frac{1}{2}k(y_f^2 - y_i^2) - mg(y_f - y_i) = 0$$

$$-\frac{1}{2}(200)(y_f^2 - 0) - (2)(10)(y_f - 0) = 0$$

$$-100y_f^2 - 20y_f = 0$$

$$-100y_f^{\cancel{2}} = 20\cancel{y_f}$$

$$\boxed{y_f = -0.2\,m}$$

More than one Spring