

# **Equations**

- Solving without numbers
- Units in expressions
- Dimensional consistency

## Solving without Numbers

$$a = \frac{4\pi^2 r}{T^2}$$

a) Solve the equation for  $r$ .

b) Solve the equation for  $T$ .

$$\frac{a}{1} = \frac{4\pi^2 r}{T^2}$$
$$aT^2 = 4\pi^2 r$$

$$\frac{aT^2}{4\pi^2} = r$$

$$\frac{a}{1} = \frac{4\pi^2 r}{T^2}$$

$$aT^2 = 4\pi^2 r$$

$$T^2 = \frac{4\pi^2 r}{a}$$

$$T = \sqrt{\frac{4\pi^2 r}{a}}$$

## Units in Expressions

$W = F(\Delta x)$  W is Work in Joules, and  $\Delta x$  is displacement in meters. Show that the units of F are Newtons.

$$\text{Joules} = (\text{Newtons}) (\text{meters})$$

$$\frac{\text{kgm}^2}{\text{s}^2} = \left( \frac{\text{kgm}}{\text{s}^2} \right) (\text{m})$$

✓

## Units in Expressions

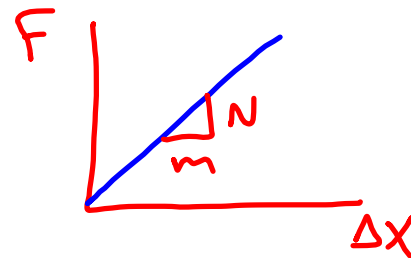
$U_e = \frac{1}{2}k(\Delta x)^2$   $U_e$  is elastic potential energy in Joules, and  $\Delta x$  is displacement in meters. What are the units of  $k$ ?

$$(\text{Joules}) = (\text{?}) (\text{m})^2$$

$$\frac{\text{kgm}^2}{\text{s}^2} = (\text{?}) \text{m}^2$$

$$\frac{\text{kgm}^2}{\text{s}^2} = \left( \frac{\text{kg}}{\text{s}^2} \right) \text{m}^2$$

$k$  is in  $\frac{\text{kg}}{\text{s}^2}$



$$k = \text{slope} \frac{\text{N}}{\text{m}}$$

$$\frac{\cancel{\text{kgm}}}{\cancel{\text{s}^2} \cancel{\text{m}}}$$

# Dimensional Consistency

All of the terms\* on both sides of an equation must have matching units.

\* terms are separated by + or - signs.

$$\underline{E_{mech}} = \underline{\frac{1}{2}mv^2} + \underline{mgh}$$

## EX: Dimensional Consistency

Determine whether the equation below is dimensionally consistent.

$$E_{\text{mech}} = \frac{1}{2}mv^2 + mgh$$

$$(\text{Joules}) = (\text{Joules}) + (\text{Joules})$$

$$\text{kg} \frac{\text{m}^2}{\text{s}^2} = (\text{kg}) \left(\frac{\text{m}}{\text{s}}\right)^2 + (\text{kg}) \left(\frac{\text{m}}{\text{s}^2}\right) (\text{m})$$

$$\text{kg} \frac{\text{m}^2}{\text{s}^2} \quad \checkmark = \text{kg} \frac{\text{m}^2}{\text{s}^2} + \text{kg} \frac{\text{m}^2}{\text{s}^2}$$